Lecture 3

2023/2024

Microwave Devices and Circuits for Radiocommunications

2023/2024

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course or laboratory)**
- Lectures- **associate professor Radu Damian**
  - Tuesday 16-18, ~~Online~~, P8
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - first test L1: 20-27.02.2024 (t2 and t3 not announced, lecture)
    - 3att.=+0.5p
  - **all materials/equipments authorized**

2023/2024

- Laboratory – **associate professor Radu Damian**
  - Tuesday 08-12, II.13 / (08:10)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + **personal results**
  - P – 25% final grade
    - ADS, 3 sessions (-1? 20.02.2024)
    - personal homework

Materials

- http://rf-opto.etti.tuiasi.ro

Laboratorul de Microunde si Opt

Not secure | rf-opto.etti.tuiasi.ro/microwave_cd.php?chg_lang=0

Main **Courses** Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

Aggregate Results

Attendance

Course
Laboratory

Lists

Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

Materials

Course Slides

MDCR Lecture 1 (pdf, 5.43 MB, en,)
MDCR Lecture 2 (pdf, 3.67 MB, en,)
MDCR Lecture 3 (pdf, 4.76 MB, en,)
MDCR Lecture 4 (pdf, 5.58 MB, en,)

RF-OPTO

English | Romana |

Main Courses Master Staff Rese

Grades Student List **Exams** Photos

Online Exams

In order to participate at online exams you must get ready following

Site

Main Courses Master Staff Research Students

Microwave and Optoelectronics Laboratory

We are enlisted in the Telecommunications Department of the Electronics, Telecommunication and Information Technology Faculty (**ETTI**) from the "Gh. Asachi" Technical University (**TUIASI**) in Iasi, Romania

We currently cover inside **ETTI** the fields related to:

- Microwave Circuits and Devices
- Optoelectronics
- Information Technology

Courses

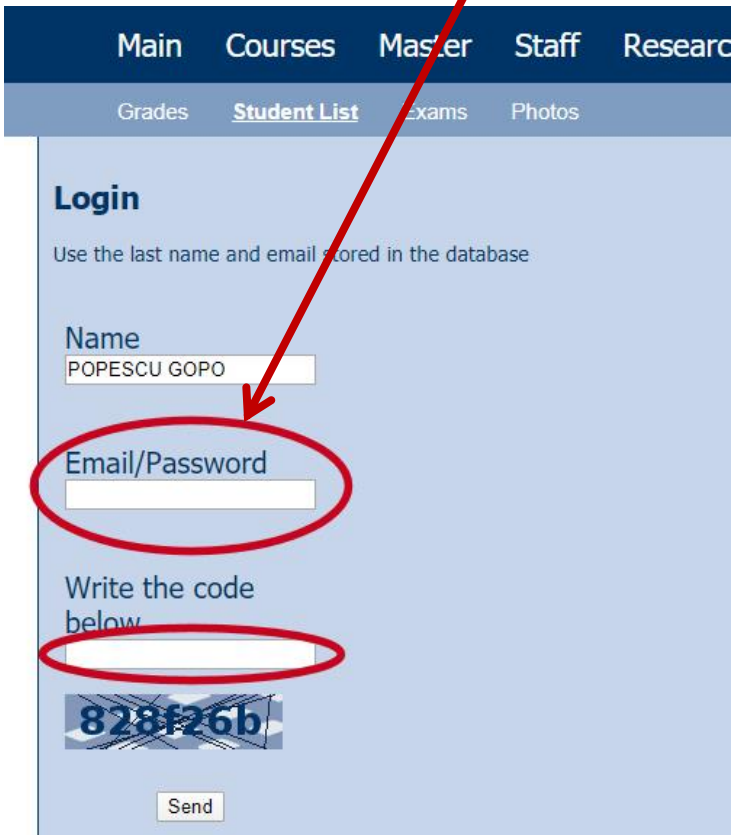
Nr.	Course	Shortcut	Code	Type	Semester	Credits	Weekly	Examination	Link
1	Microwave Devices and Circuits for Radiocommunications	**DCMR**	DOS412T	DOS	7	4	0P,1L,0S,2C	Exam	
2	Monolithic Microwave Integrated Circuits	**CIMM**	RD.IA.207	DOMS	11	6	1.5L,0S,2C,0P	Exam	
3	Advanced Techniques in the Design of the Radio-communications Systems	**TAPSR**	RD.IA.103	DIMS	9	6	1.5P,0L,0S,2C	Exam	
4	Optical Communications	**CO**	DOS409T	DOS	7	5	0P,1L,0S,3C	Colloquiu	
5	Optical Communications	**OC**	EDOS409T	DOS	7	5	0P,1L,0S,3C	Exam	
6	Satellite Communications	**CS**	RC.IA.104	DIMS	9	6	0L,0S,2C,1.5P	Exam	
7	Applied Informatics 1	**IA1**	DOF135	DOF	1	4	0P,1L,0S,2C	Verificati	
8	Applied Informatics 1	**AI1**	EDOF135	DOF	1	4	0P,1L,0S,2C	Verificati	
9	Databases, Web Programming and Interfacing	**DWPI**	ITT.IA.601	DIS	11	5	1P,1L,0.25S,1C	Verificati	
10	Web Applications Design	**PAW**	RC.IA.108	DIMS	10	5	1L,0S,1.5C,1P	Exam	
11	Optoelectronics	**OPTO**	DID405M	DID	8	4	0P,1L,0S,2C	Colloquiu	
12	Microwave Devices and Circuits for Radiocommunications (English)	**MDCR**	EDOS412T	DOS	8	4	0P,1L,0S,2C	Exam	

Materials

- RF-OPTO
  - **http://rf-opto.etti.tuiasi.ro**
- **David Pozar, "Microwave Engineering"**, Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/**online exam > Week4-Week6**
  - used at lectures/laboratory

Online

- access to **online exams** requires the **password** received by email



Password

- received by email

Important message from RF-OPTO Inbox ×

Radu-Florin Damian
to me, POPESCU

Romanian > English Translate message

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
Universitatea Tehnica "Gh. Asachi" Iasi

In atentia: POPESCU GOPO ION

Parola pentru a accesa examenele pe server-ul **rf-opto** este
Parola: [redacted]

Identificati-va pe server, cu parola, cat mai rapid, pentru confirmare.

Memorati acest mesaj intr-un loc sigur, pentru utilizare ulterioara

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Attention: POPESCU GOPO ION

The password to access the exams on the **rf-opto** server is
Password: [redacted]

Login to the server, with this password, as soon as possible, for confirmation.

Save this message in a safe place for later use

Reply Reply all Forward

Subject	Correspondents
Important message from RF-OPTO	POPESCU GOPO ION
Validation of MDCR exam from 02/05/2020	[redacted]
[redacted]	[redacted]

From Me <rdamian@etti.tuiasi.ro>
Subject **Important message from RF-OPTO**
To [redacted]
Cc Me <rdamian@etti.tuiasi.ro>

Laboratorul de Microunde si Optoelectronica
Facultatea de Electronica, Telecomunicatii si Tehnologia Informatiei
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In atentia: POPESCU GOPO ION

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Password: [redacted]

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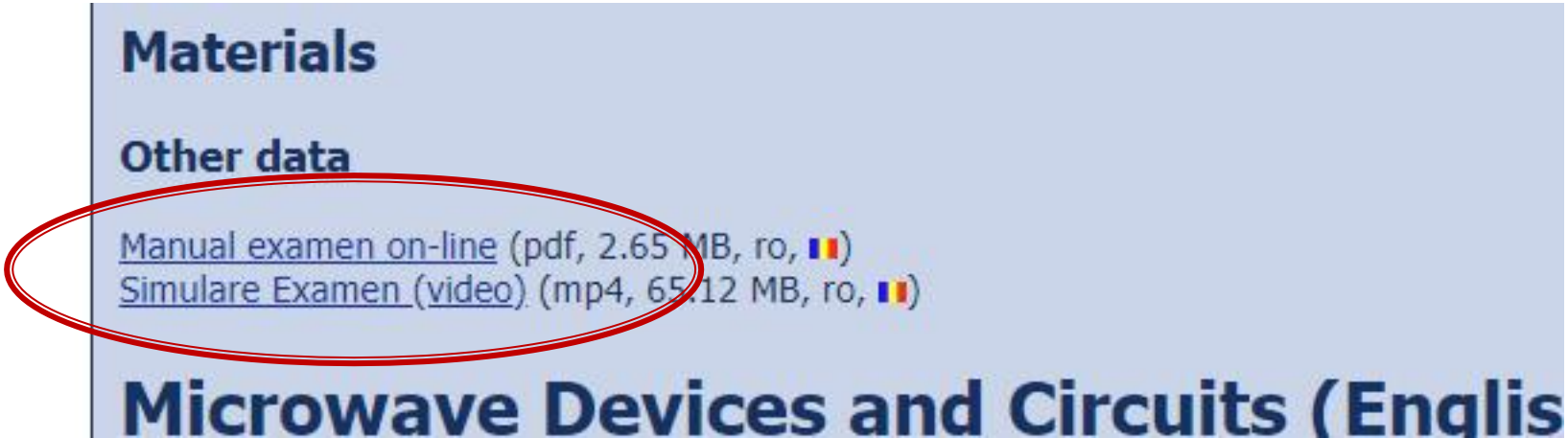
Save this message in a safe place for later use

Adrese email

- Sefii de grupa
  - lista cu adrese de email **utilizate** de toti studentii
    - poate fi @student.etti.tuiasi.ro (@gmail @yahoo etc.)
  - **rdamian@etti.tuiasi.ro**

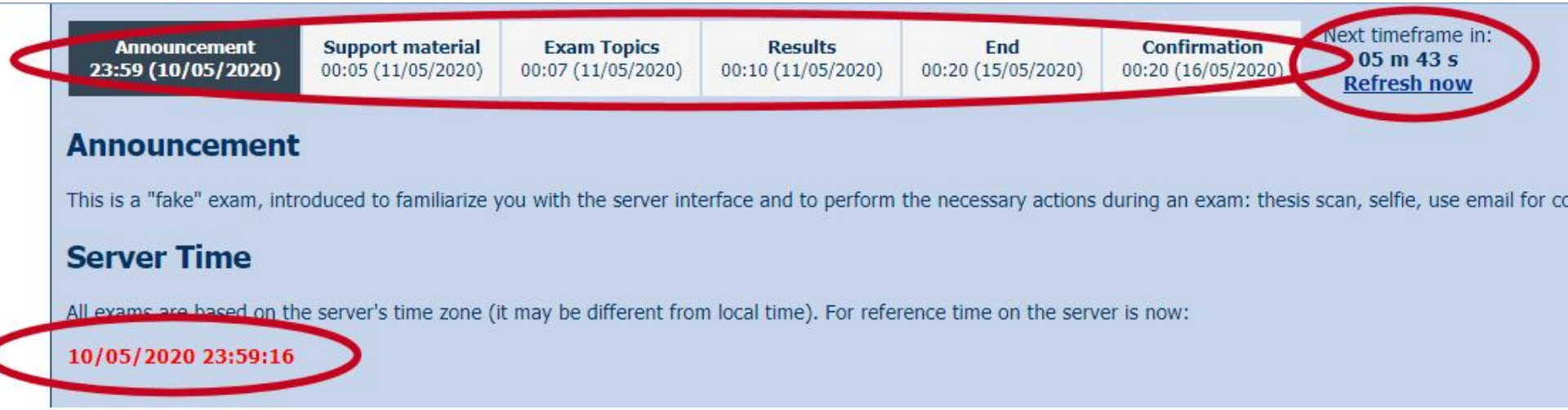
Online exam manual

- The online exam app used for:
  - ~~lectures (attendance)~~
  - laboratory
  - project
  - ~~examinations~~



Examen online

- always against a **timetable**
  - long period (lecture attendance/laboratory results)
  - ~~short period (tests: 15min, exam: 2h)~~



Online results submission

- many numerical values/files

Schema finala	Rezultate - castig	Rezultate - zgomot	Fisier justificare calcul (factor andrei)	Fisier zap (optional)	T1, fisier parmetri S	T2, fisier parmetri S	Z1	Z2	Z3	Z4	Z5	Z6	Z7	Ze1	Zo1	Ze2	Zo2	Ze3	Zo3	Ze4	Zo4	Ze5	Zo5	Ze6
86 - 5428 - 259 ...	86 - 5428 - 260 ...	86 - 5428 - 261 ...	86 - 5428 - 316 ...	-	86 - 5428 - 314 ...	86 - 5428 - 315 ...	148.33	155.88	202.12	164.35	180.91	30.29	185.19	79.9	37	68.89	45.14	61.83	45.05	57.97	46.02	61.85	45.05	68.8
86 - 5622 - 259 ...	86 - 5622 - 260 ...	86 - 5622 - 261 ...	86 - 5622 - 316 ...	86 - 5622 - 262 ...	86 - 5622 - 314 ...	86 - 5622 - 315 ...	26.97	153.5	34.64	35.79	55.56	26.212	10.693	0	0	0	0	0	0	0	0	0	0	0
86 - 5488 - 259 ...	86 - 5488 - 260 ...	86 - 5488 - 261 ...	86 - 5488 - 316 ...	86 - 5488 - 262 ...	86 - 5488 - 314 ...	86 - 5488 - 315 ...	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
86 - 5391 - 259 ...	86 - 5391 - 260 ...	86 - 5391 - 261 ...	86 - 5391 - 316 ...	-	-	-	50	50	50	50	50	50	50	70.14	40.39	61.85	44.59	55.7	45.2	54.89	45.38	58.65	45.8	70.0
86 - 5664 - 259 ...	86 - 5664 - 260 ...	86 - 5664 - 261 ...	86 - 5664 - 316 ...	-	86 - 5664 - 314 ...	86 - 5664 - 315 ...	168.02	150.5	178.28	133.75	92.12	121.67	144.48	94.36	36.19	70.77	42.56	65.69	42.05	55.17	42.29	65.59	42.05	70.7
86 - 5665 - 259 ...	86 - 5665 - 260 ...	86 - 5665 - 261 ...	86 - 5665 - 316 ...	-	86 - 5665 - 314 ...	86 - 5665 - 315 ...	162.2	80.8	209.2	140.85	135.1	183.7	167.6	94.58	36.15	78.16	39.77	65.57	45.05	65.57	45.05	78.16	39.77	94.5
86 - 5433 - 259 ...	86 - 5433 - 260 ...	86 - 5433 - 261 ...	86 - 5433 - 316 ...	-	86 - 5433 - 314 ...	86 - 5433 - 315 ...	165.138	106.228	226.157	130.134	72.71	180.177	164.616	101.36	36.11	77.22	42.49	68.02	45.62	60	45.42	68.02	45.62	77.2
86 - 5608 - 259 ...	86 - 5608 - 260 ...	86 - 5608 - 261 ...	86 - 5608 - 316 ...	-	86 - 5608 - 314 ...	86 - 5608 - 315 ...	150.84	152.5	30.94	32.37	54.36	19.837	29.85	64.14	40.145	54.32	46.32	53.8	46.7	53.8	46.7	54.32	46.32	54.9
86 - 5555 - 259 ...	86 - 5555 - 260 ...	86 - 5555 - 261 ...	86 - 5555 - 316 ...	-	86 - 5555 - 314 ...	86 - 5555 - 315 ...	168.001	150.288	178.399	133.115	92.491	121.257	144.126	97.05	36.16	71.13	43.09	65.45	42.12	55.66	42.18	65.45	42.12	71.1

Online results submission

- many numerical values

Z1	Z2	Z3	Z4	Z5	Z6	Z7
148.33	155.88	202.12	164.35	180.91	30.29	185.19
26.97	153.5	34.64	35.79	55.56	26.212	10.693
0	0	0	0	0	0	0
50	50	50	50	50	50	50

√

✗

Online results submission

Grade = Quality of the work +

+ Quality of the submission

Computing Loss/Gain in circuits

$$\text{Loss} = \frac{P_{out}}{P_{in}} < 1$$

$$\text{Loss[dB]} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) < 0$$

$$\text{Loss/Attenuation[dB]} = [-]\ 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right)$$

$$\text{Gain} = \frac{P_{out}}{P_{in}} > 1$$

$$\text{Gain[dB]} = 10 \cdot \log_{10}\left(\frac{P_{out}}{P_{in}}\right) > 0$$

$$\text{Attenuation[dB/km]} = \frac{\text{Loss[dB]}}{\text{Length[km]}}$$

Computing Loss/Gain in circuits

$$\text{Loss/Attenuation} \rightarrow P_{out} < P_{in} \rightarrow P_{out}[\text{dBm}] < P_{in}[\text{dBm}]$$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] - \text{Loss/Attenuation}[\text{dB}]$$

$$\text{Gain/Amplification} \rightarrow P_{out} > P_{in} \rightarrow P_{out}[\text{dBm}] > P_{in}[\text{dBm}]$$

$$P_{out}[\text{dBm}] = P_{in}[\text{dBm}] + \text{Gain/Amplification}[\text{dB}]$$

Exam: Logarithmic scales

dB = 10 • $\log_{10}$ (P_2 / P_1)

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

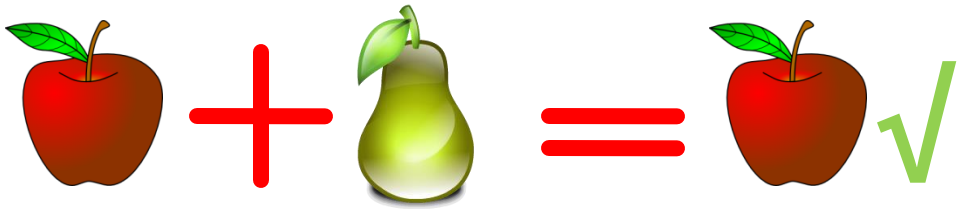
dBm = 10 • $\log_{10}$ (P / 1 mW)

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 µW
-30 dBm	= 1 µW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

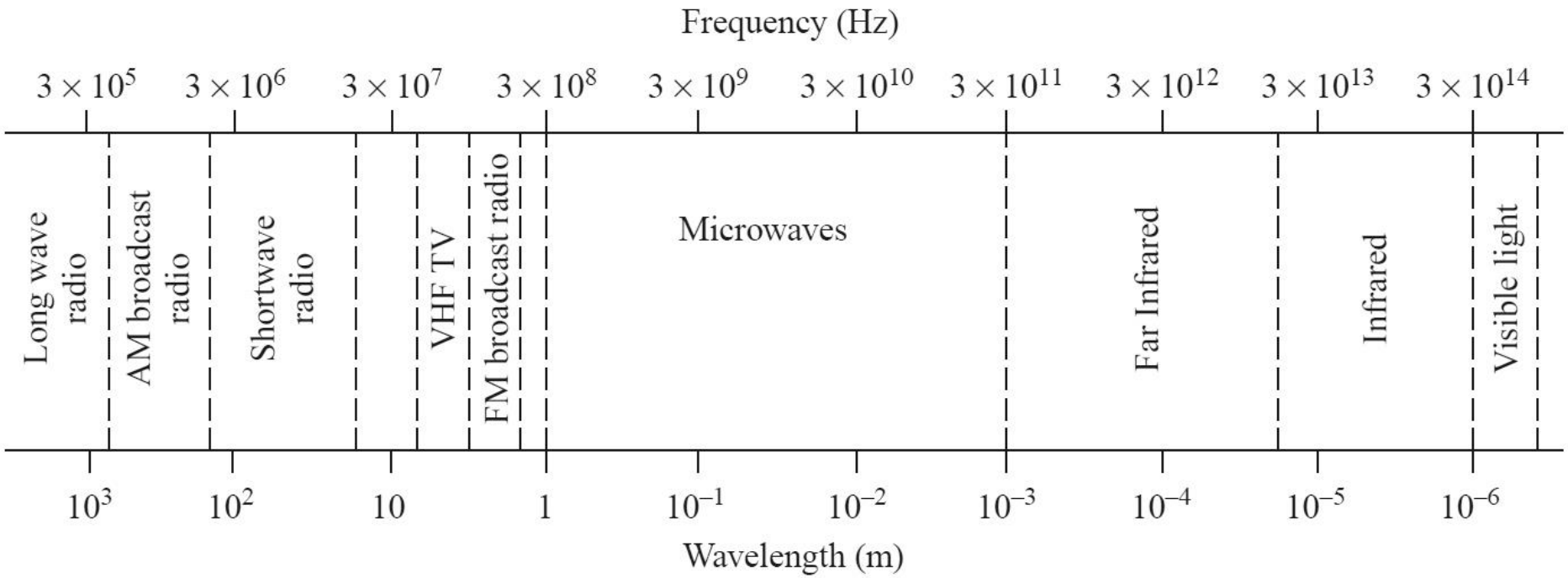
[x] + [dB] = [x]

[dBm/Hz] + [dB] = [dBm/Hz]



Introduction

Microwaves

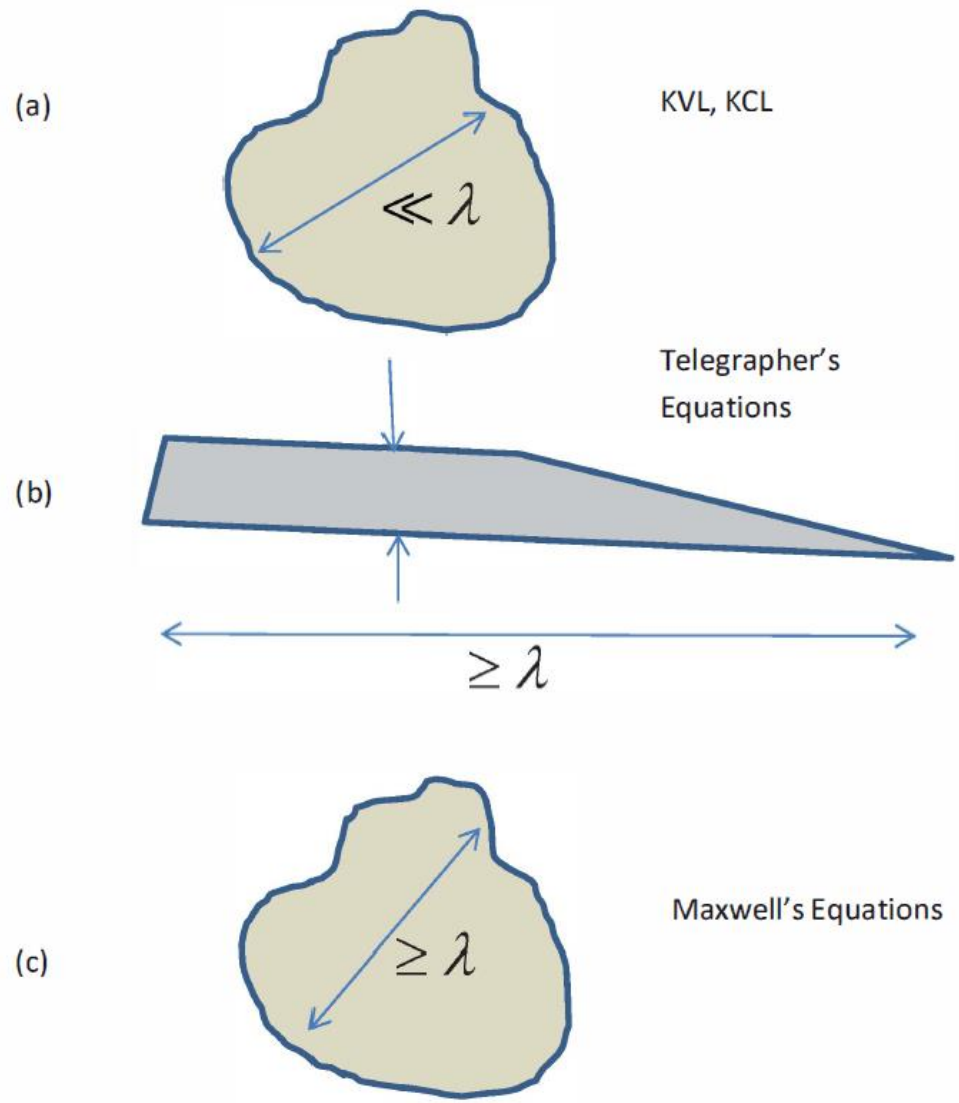


- typically
  - f ≈ 1÷3GHz – 300GHz
  - λ ≈ 1mm – 10cm

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈0 → Kirchhoff
  - E>0 → wave propagation

$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda} \right)$$



Maxwell's Equations

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

- Constitutive equations

$$D = \varepsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

- Vacuum

$$\mu_0 = 4\pi \times 10^{-7}\ H/m$$

$$\varepsilon_0 = 8{,}854 \times 10^{-12}\ F/m$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2{,}99790 \cdot 10^8\ m/s$$

Wave equations

- Helmoltz equations or Wave equations

Medium void of free electric charges

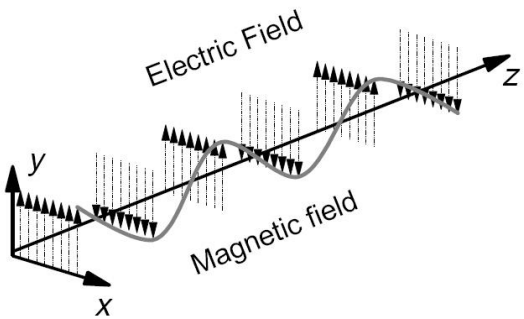
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

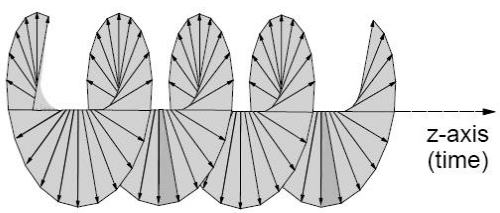
$$\gamma^2 = -\omega^2 \varepsilon\mu + j\omega\mu\sigma$$

γ – propagation constant (known also as phase constant or wave number)

Solutions of the wave equations



Wave Propagation



Circular Polarization

Electric field only in Oy direction, ← through judicious choice
wave traveling after Oz direction ← of the coordinate system

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \varepsilon \mu + j\omega\mu\sigma} = \alpha + j \cdot \beta$$

If we have only the positive direction wave E_+=> A

$$E_y = Ae^{-(\alpha + j \cdot \beta) \cdot z}$$

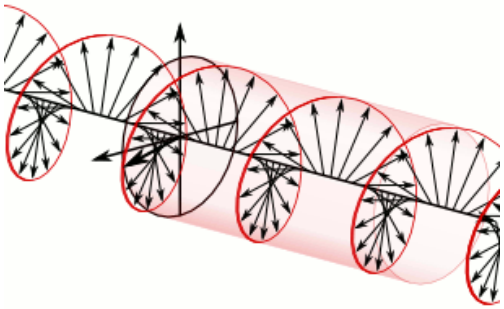
Harmonic Field

$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$$

Amplitude

Attenuation

Wave Propagation (simultaneous space and time variation)



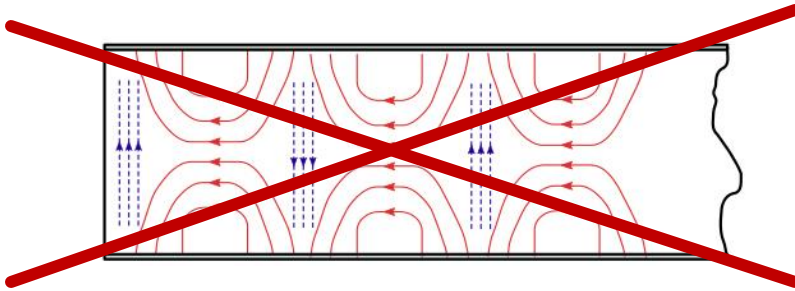
TEM transmission lines

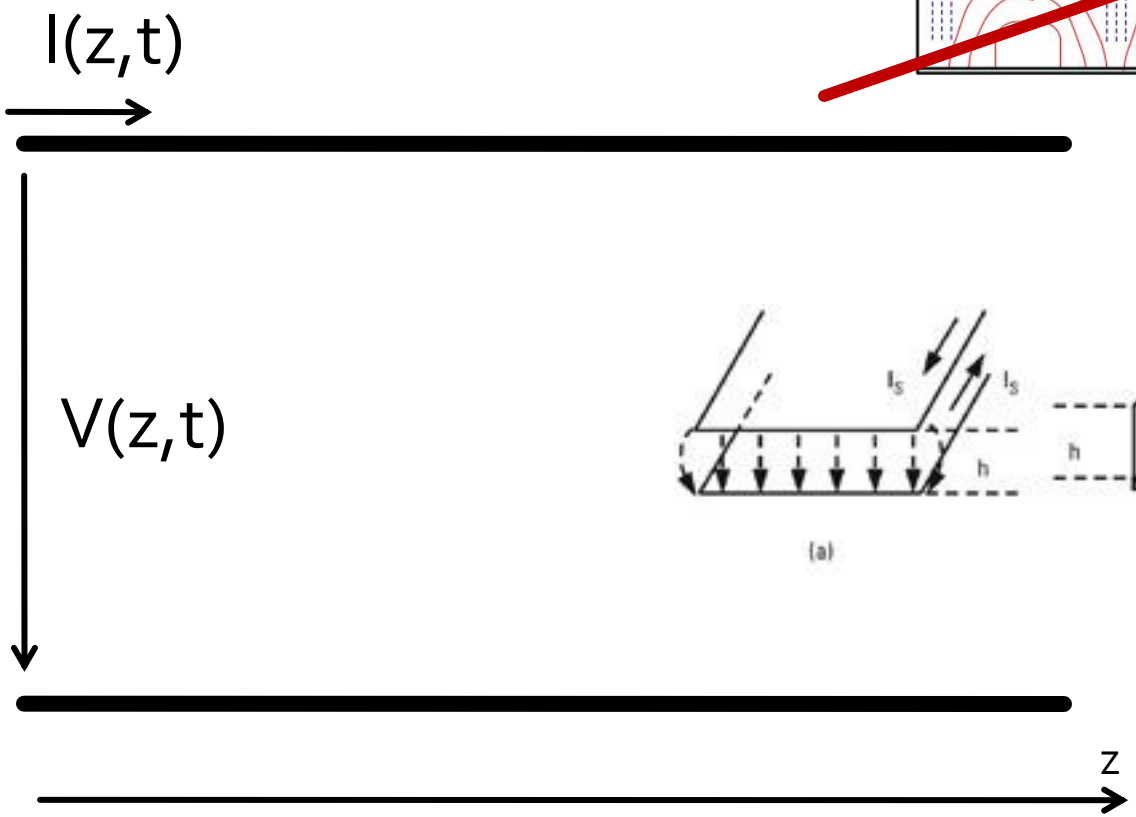
Course Topics

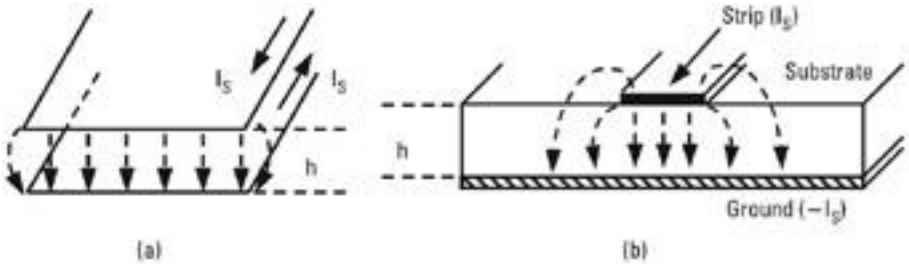
- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Transmission line

- TEM wave propagation, at least two conductors

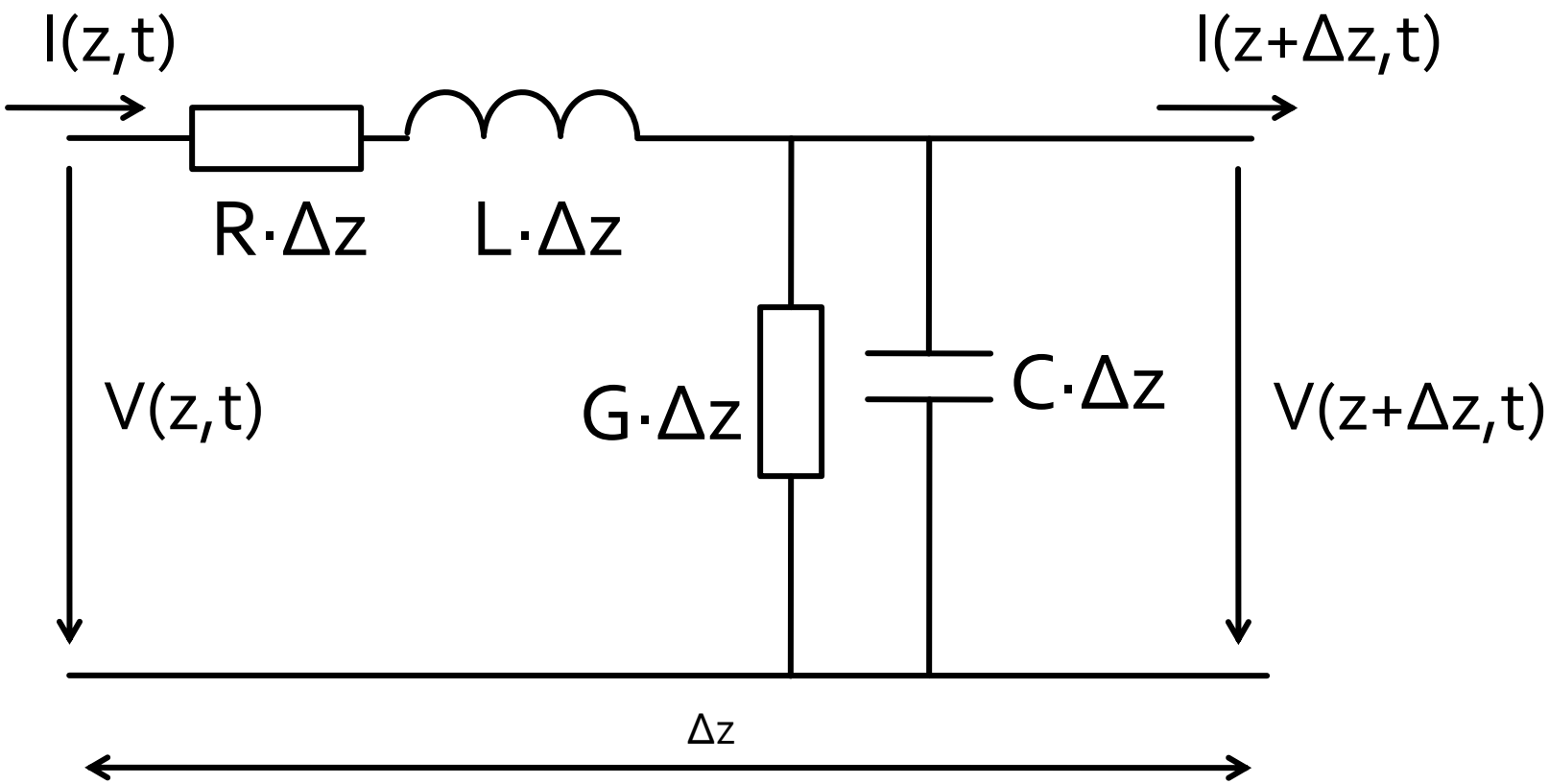






Transmission line equivalent model

- TEM wave propagation, at least two conductors



- **distributed** (line) parameters R, L, G, C (eg. Ω/m)

Telegrapher's equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \qquad \text{K II}$$

$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \qquad \text{K I}$$

- armonic signals (frequency domain)

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

$$\Big/ \frac{d}{dz}(\ldots)$$

Solving T's E

$$\frac{d^2V(z)}{dz^2} - \gamma^2 \cdot V(z) = 0$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$\frac{d^2I(z)}{dz^2} - \gamma^2 \cdot I(z) = 0$$

$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma^2 = -\omega^2 \varepsilon \mu + j \omega \mu \sigma$$

Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{cases}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} \left(V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z} \right)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- **Lossless**: R=G=0

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)} = j \cdot \omega \cdot \sqrt{L \cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega \cdot \sqrt{L \cdot C}$$

$$Z_0 = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_o is **real**

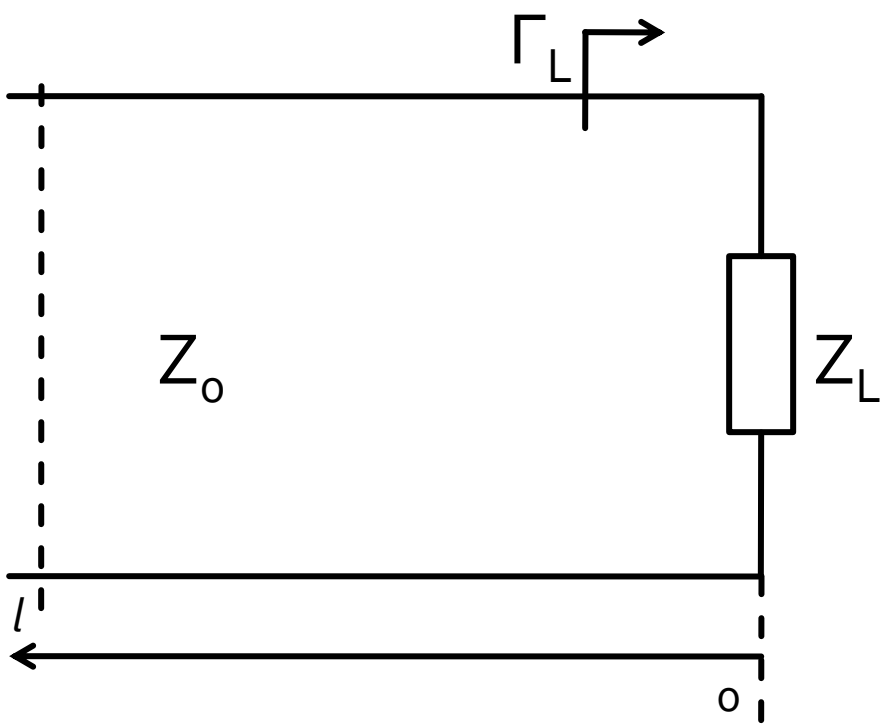
$$V(z) = V_0^+ e^{-j \cdot \beta \cdot z} + V_0^- e^{j \cdot \beta \cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j \cdot \beta \cdot z} - \frac{V_0^-}{Z_0} e^{j \cdot \beta \cdot z}$$

$$\lambda = \frac{2\pi}{\omega \cdot \sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

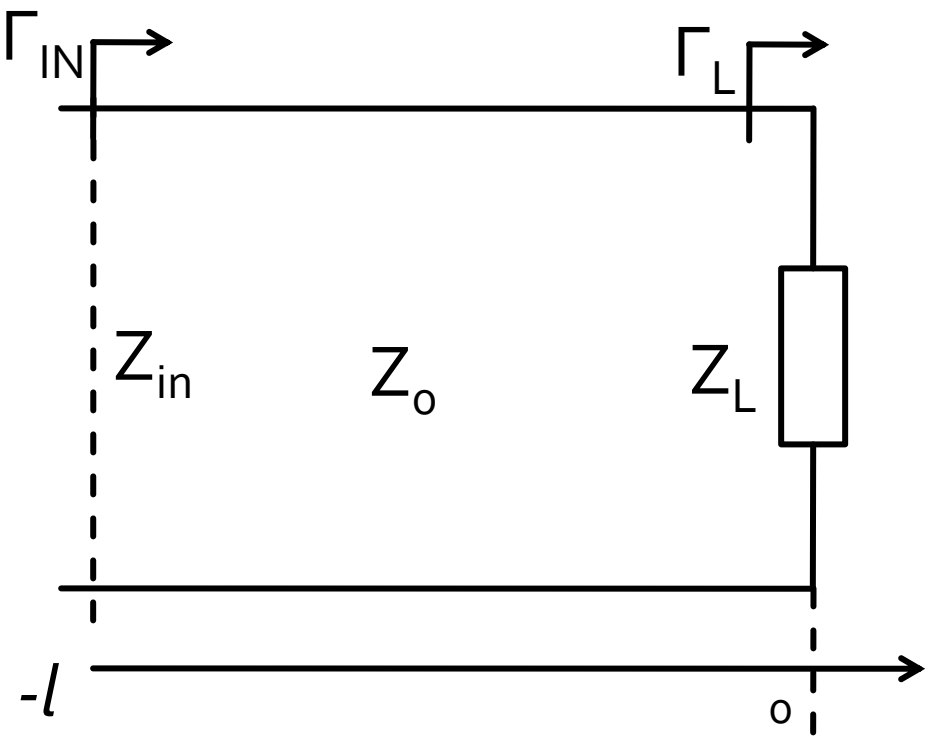
- Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z)=V_0^+ e^{-j\cdot\beta\cdot z}+V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma=\Gamma(z)=\frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0)=V_0^+ + V_0^-$$

$$\Gamma(0)=\Gamma_L=\frac{V_0^-}{V_0^+}$$

$$V(-l)=V_0^+ e^{j\cdot\beta\cdot l}+V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l)=\Gamma_{IN}=\frac{V_0^-\cdot e^{-j\cdot\beta\cdot l}}{V_0^+\cdot e^{j\cdot\beta\cdot l}}=\Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)|=|\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right|=|\Gamma(0)|$$

$$\boxed{\Gamma_{IN}=\Gamma_L\cdot e^{-2j\cdot\beta\cdot l}}$$

$$\boxed{|\Gamma_{IN}|=|\Gamma_L|}$$

The lossless line

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z) = \frac{V_0^+}{Z_0} \cdot \left(e^{-j\cdot\beta\cdot z} - \Gamma \cdot e^{j\cdot\beta\cdot z}\right)$$

- time-average Power flow along the line

$$P_{avg} = \frac{1}{2} \cdot \mathrm{Re}\left\{V(z) \cdot I(z)^*\right\} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \mathrm{Re}\left\{1 - \Gamma^* \cdot e^{-2j\cdot\beta\cdot z} + \Gamma \cdot e^{2j\cdot\beta\cdot z} - \left|\Gamma\right|^2\right\}$$

$$\left(z - z^*\right) = \mathrm{Im}$$

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

- Total power delivered to the load = Incident power – "Reflected" power
- Return "Loss" [dB]

$$\mathrm{RL} = -20 \cdot \log\left|\Gamma\right| \quad [\mathrm{dB}]$$

The lossless line

$$P_{avg} = \frac{1}{2} \cdot \frac{\left|V_0^+\right|^2}{Z_0} \cdot \left(1 - \left|\Gamma\right|^2\right)$$

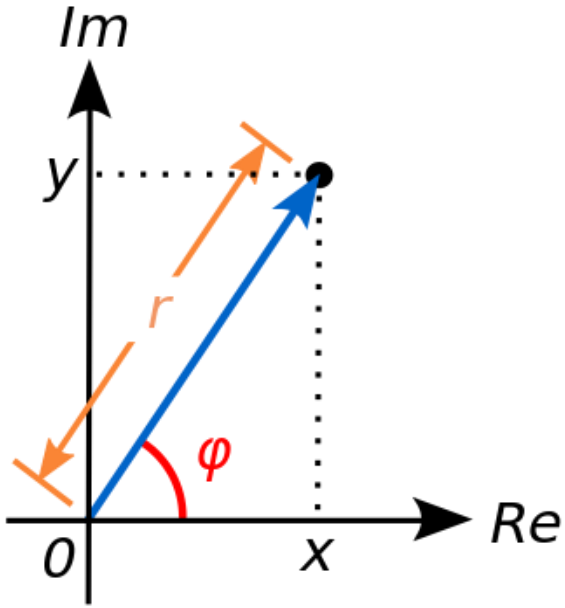
- Average power flow is constant along the line
  - (**no** P_{avg}(**z**))
  - can be measured
- We can use the power to characterize the amplitude of a signal
  - a very "energetic" (basic physics) point of view
  - more power = "more" signal

Polar representation

- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot \sin x; \forall x \in R$$

- Polar representation



$$z = a + j\cdot b = |z|\cdot e^{j\cdot\varphi}$$

$$z = a + j\cdot b = |z|\cdot(\cos\varphi + j\cdot\sin\varphi)$$

$$z^n = \left(|z|\cdot e^{j\cdot\varphi}\right)^n = |z|^n\cdot e^{j\cdot n\cdot\varphi} = |z|^n\cdot[\cos(n\cdot\varphi) + j\cdot\sin(n\cdot\varphi)]$$

$$\sqrt{z} = \left(|z|\cdot e^{j\cdot\varphi}\right)^{1/2} = \sqrt{|z|}\cdot e^{j\cdot\frac{\varphi}{2}} = \sqrt{|z|}\cdot\left(\cos\frac{\varphi}{2} + j\cdot\sin\frac{\varphi}{2}\right)$$

$$z\cdot w = |z|\cdot e^{j\cdot\varphi}\cdot|w|\cdot e^{j\cdot\theta} = |z|\cdot|w|\cdot e^{j\cdot(\varphi+\theta)} = |z|\cdot|w|\cdot[\cos(\varphi+\theta) + j\cdot\sin(\varphi+\theta)]$$

$$z/w = \frac{|z|\cdot e^{j\cdot\varphi}}{|w|\cdot e^{j\cdot\theta}} = \frac{|z|}{|w|}\cdot e^{j\cdot\varphi}\cdot e^{-j\cdot\theta} = \frac{|z|}{|w|}\cdot[\cos(\varphi-\theta) + j\cdot\sin(\varphi-\theta)]$$

Polar representation

- Euler's formula

$$e^{j\cdot x} = \cos x + j\cdot \sin x; \forall x \in R$$

$$e^{j\cdot x} + e^{-j\cdot x} = \cos x + j\cdot \sin x + \cos(-x) + j\cdot \sin(-x)$$

$$e^{j\cdot x} + e^{-j\cdot x} = \cos x + j\cdot \sin x + \cos x - j\cdot \sin x = 2\cdot \cos x$$

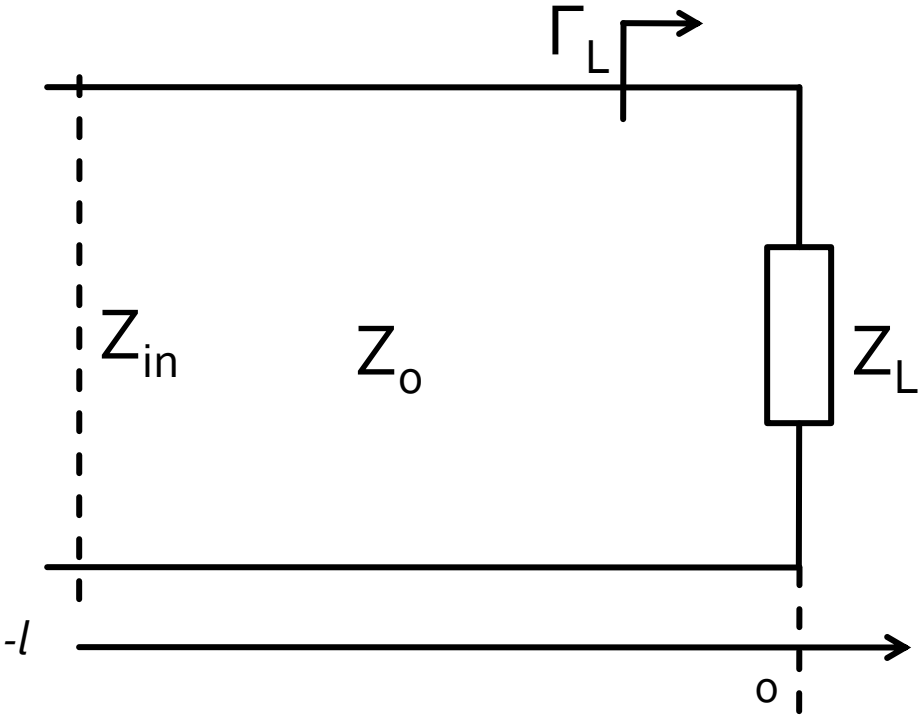
$$\rightarrow \quad \cos x = \frac{e^{j\cdot x} + e^{-j\cdot x}}{2}$$

$$e^{j\cdot x} - e^{-j\cdot x} = \cos x + j\cdot \sin x - \cos(-x) - j\cdot \sin(-x)$$

$$e^{j\cdot x} - e^{-j\cdot x} = \cos x + j\cdot \sin x - \cos x + j\cdot \sin x = 2j\cdot \sin x$$

$$\rightarrow \quad \sin x = \frac{e^{j\cdot x} - e^{-j\cdot x}}{2j}$$

The lossless line



$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\cdot\beta\cdot l} - \frac{V_0^-}{Z_0} e^{-j\cdot\beta\cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \qquad Z_{in} = Z_0 \cdot \frac{1+\Gamma\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma\cdot e^{-2j\cdot\beta\cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} + (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}{(Z_L + Z_0)\cdot e^{j\cdot\beta\cdot l} - (Z_L - Z_0)\cdot e^{-j\cdot\beta\cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0\cdot\tan(\beta\cdot l)}{Z_0 + j\cdot Z_L\cdot\tan(\beta\cdot l)}$$

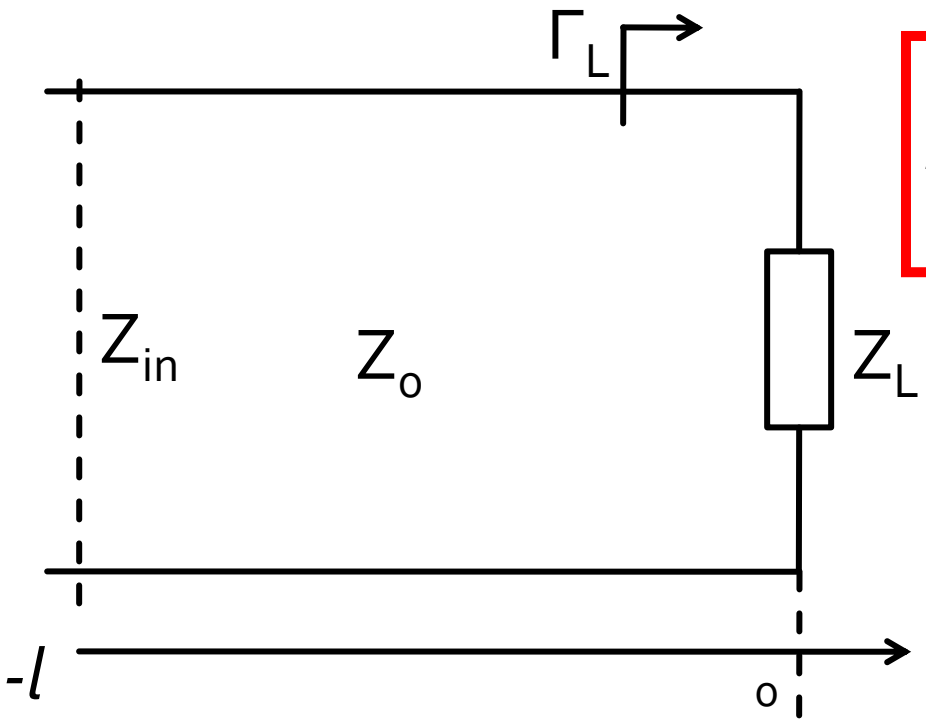
The lossless line

- the **input impedance** seen looking toward the load

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line

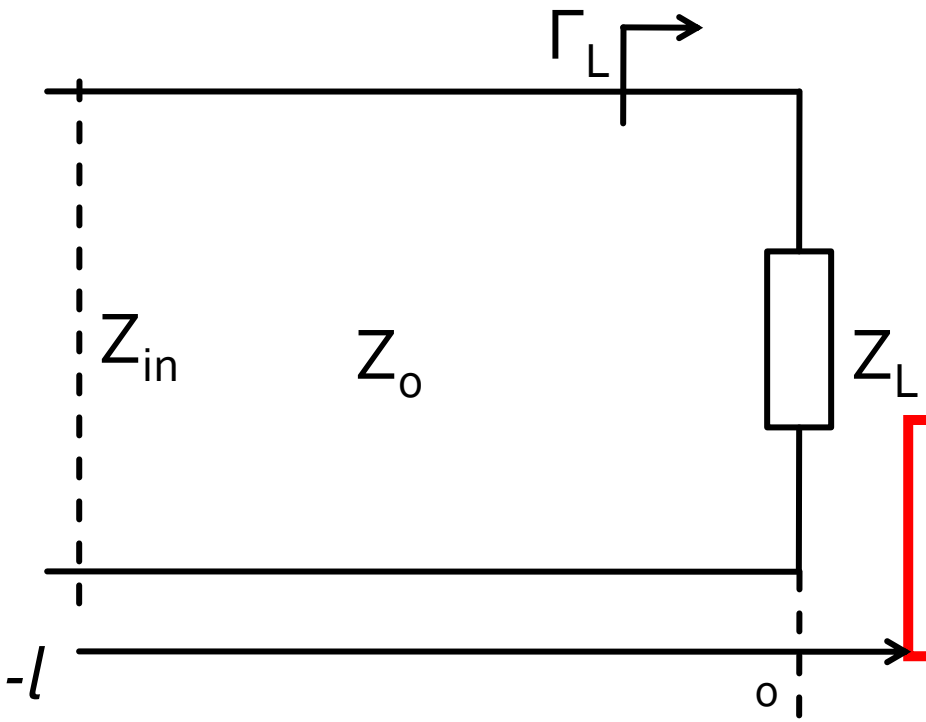
- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan\beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan\beta \cdot l}}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

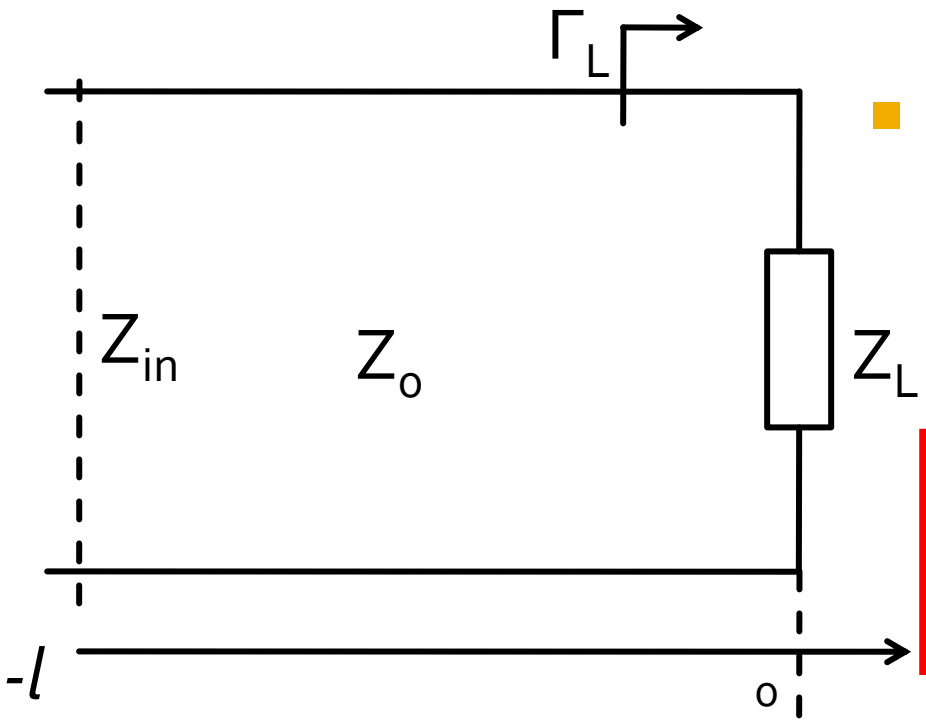
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line, special cases

- $l = k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0$ $\quad \boxed{Z_{in} = Z_L}$
- $l = \lambda/4 + k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{\pi}{2} + k \cdot \pi \quad \tan \beta \cdot l \to \infty$ $\quad \boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$



- quarter-wave transformer

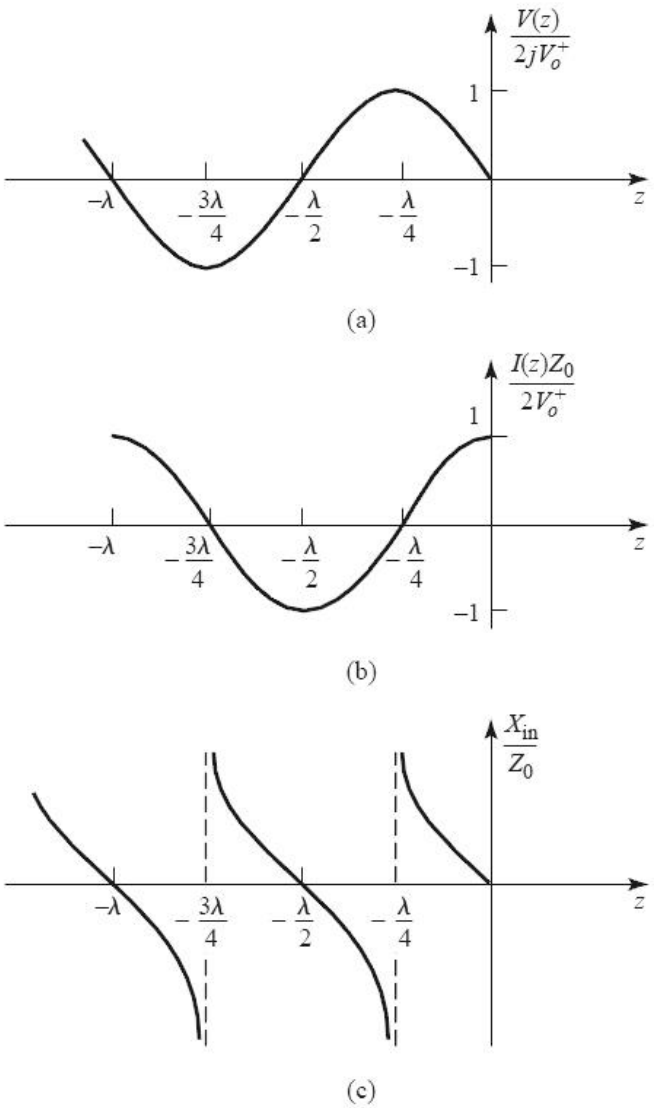
$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

Short-circuited transmission line

- $Z_L = 0$
- input purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

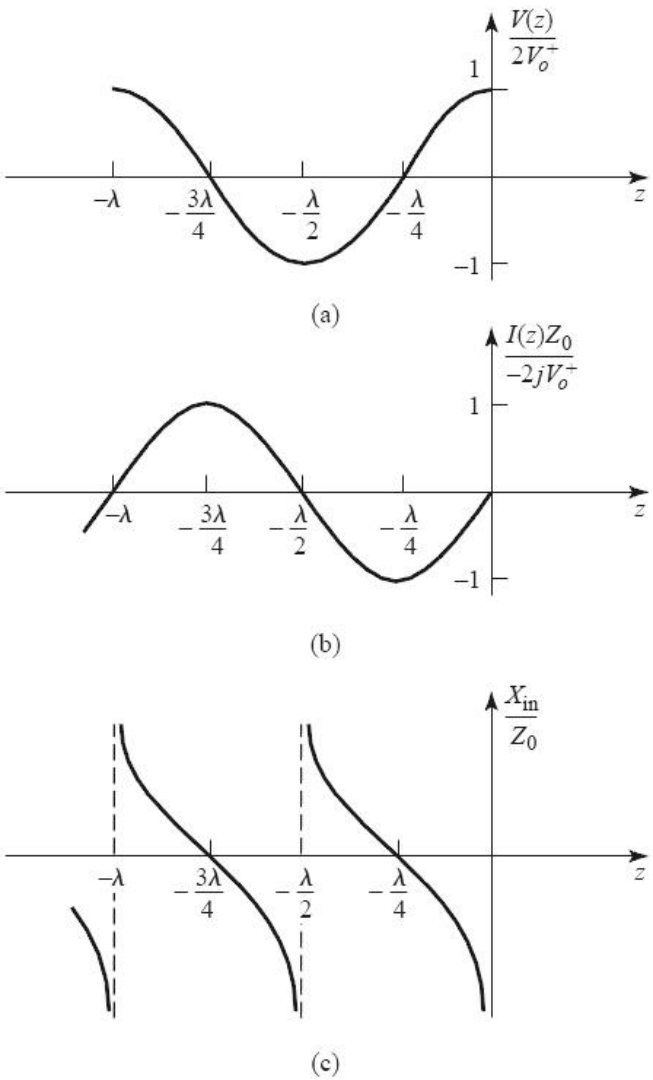


Open-circuited transmission line

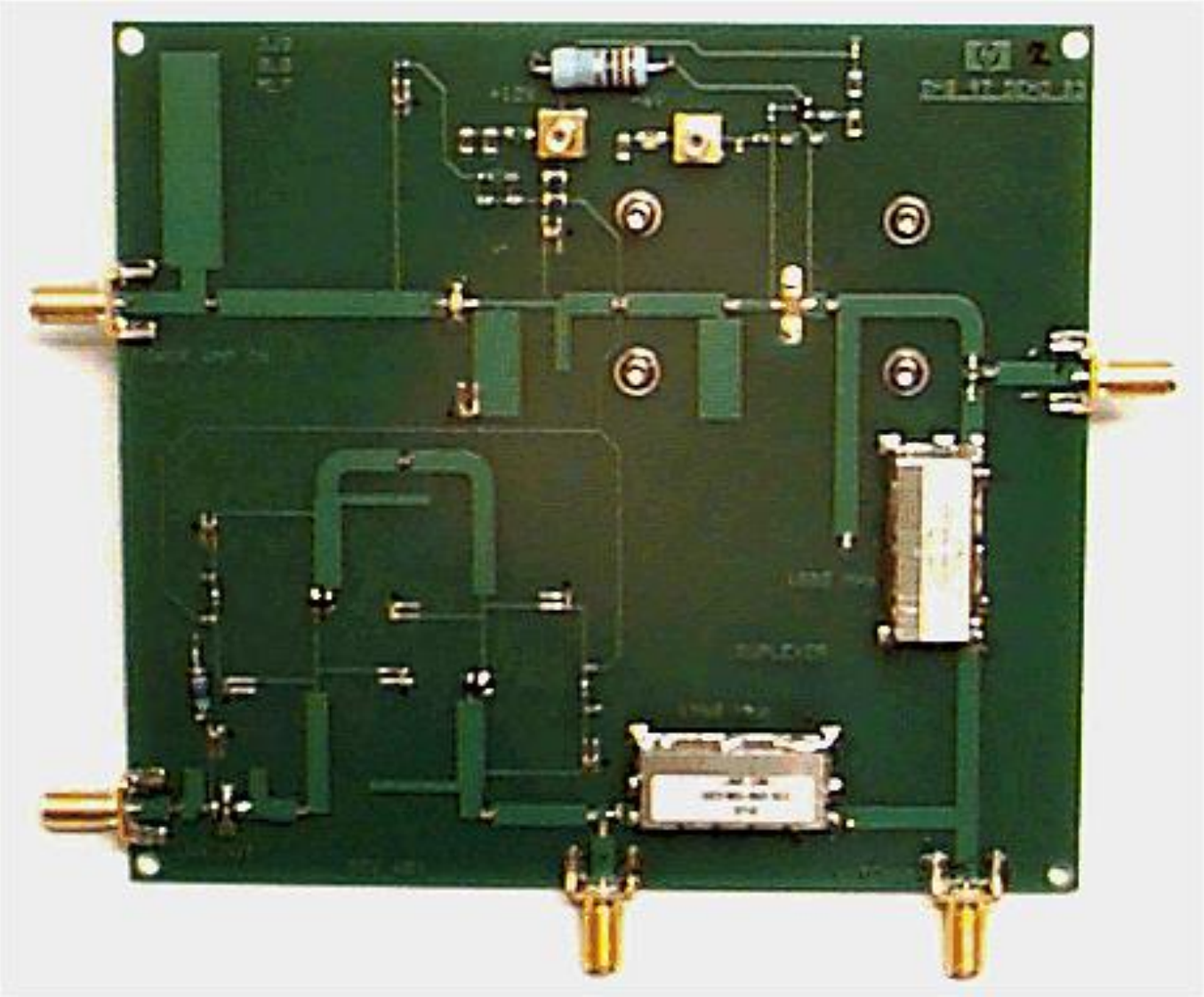
- $Z_L = \infty \longrightarrow 1 / Z_L = 0$
- input purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

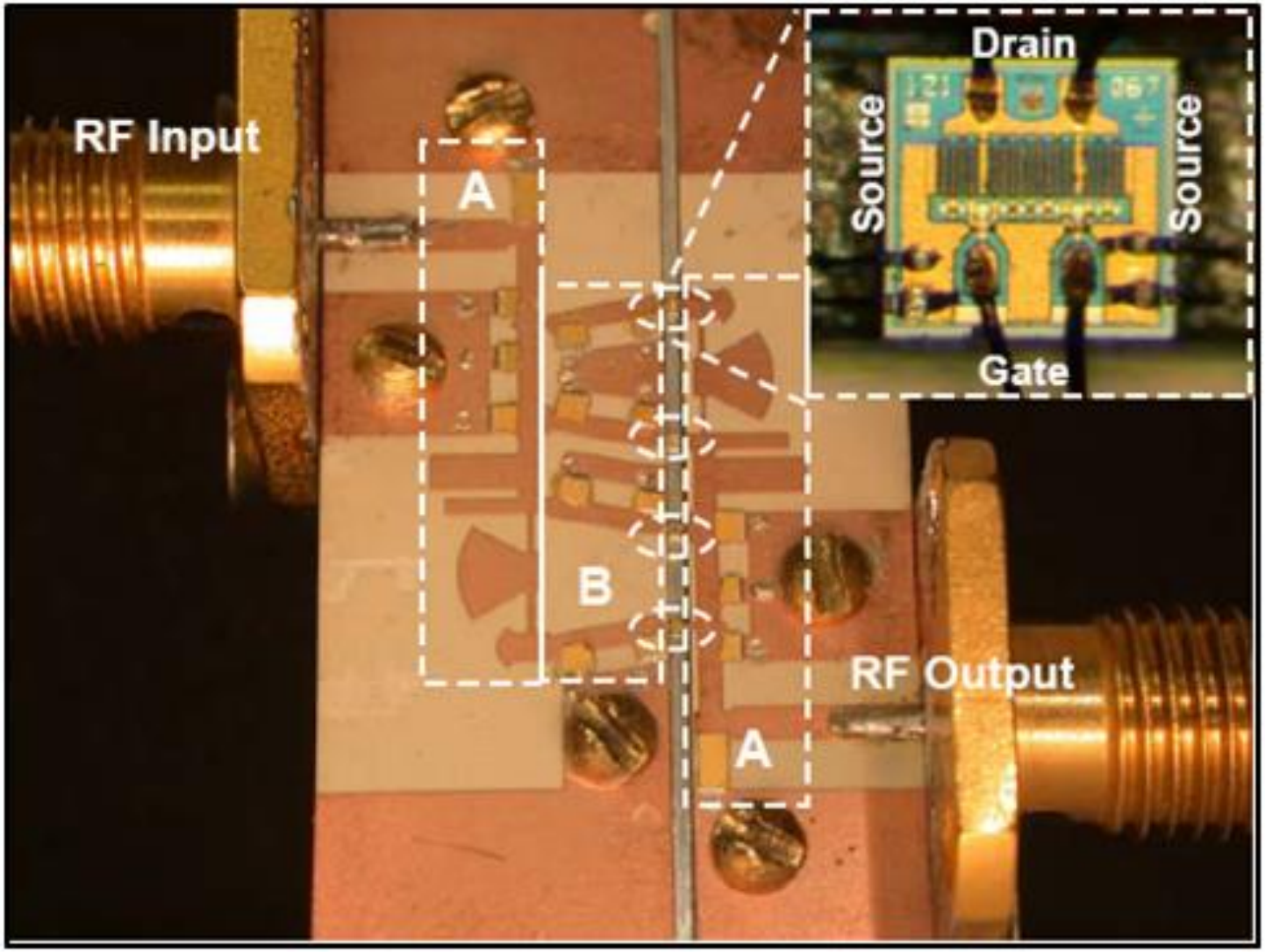
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



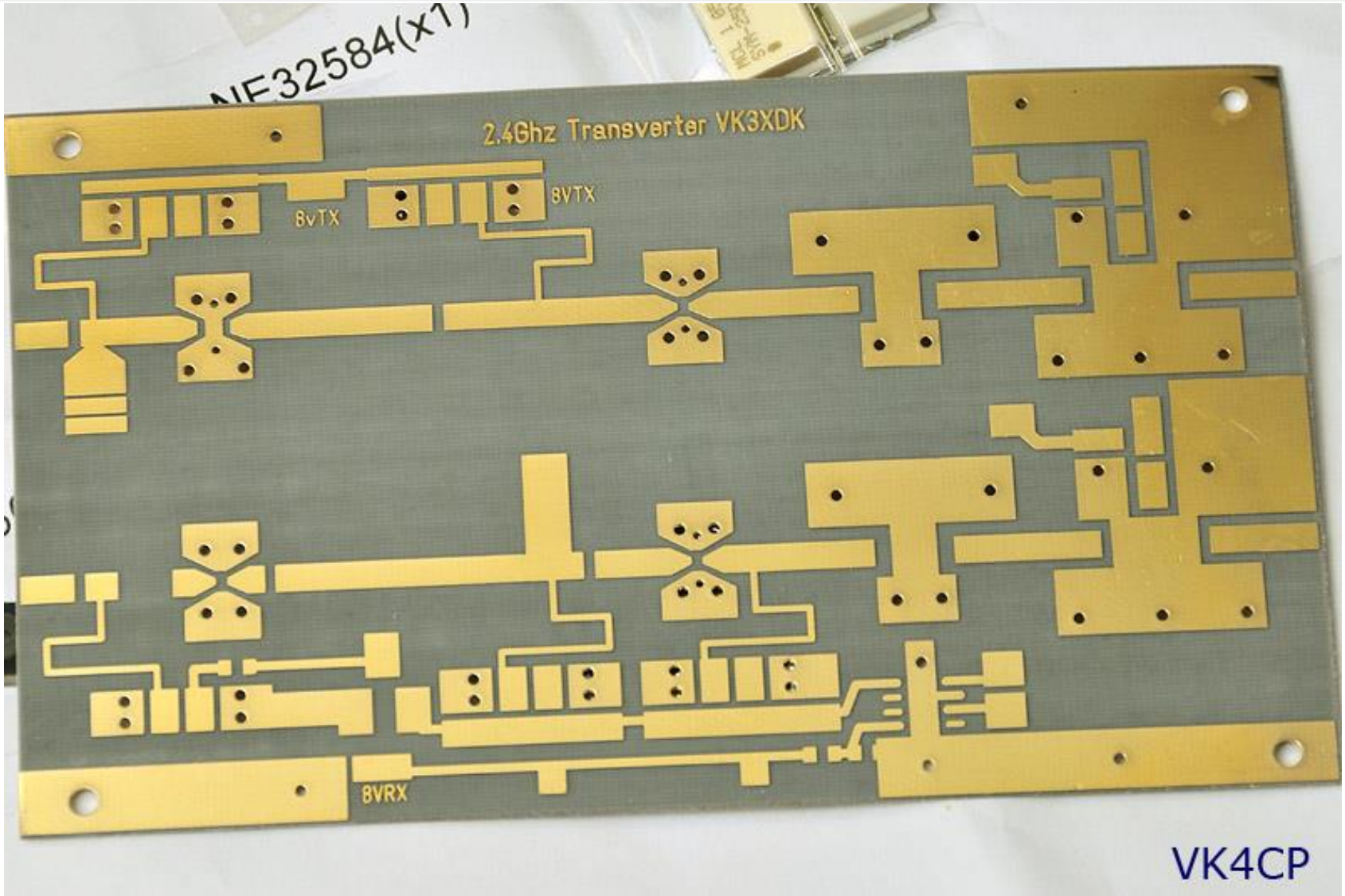
Examples



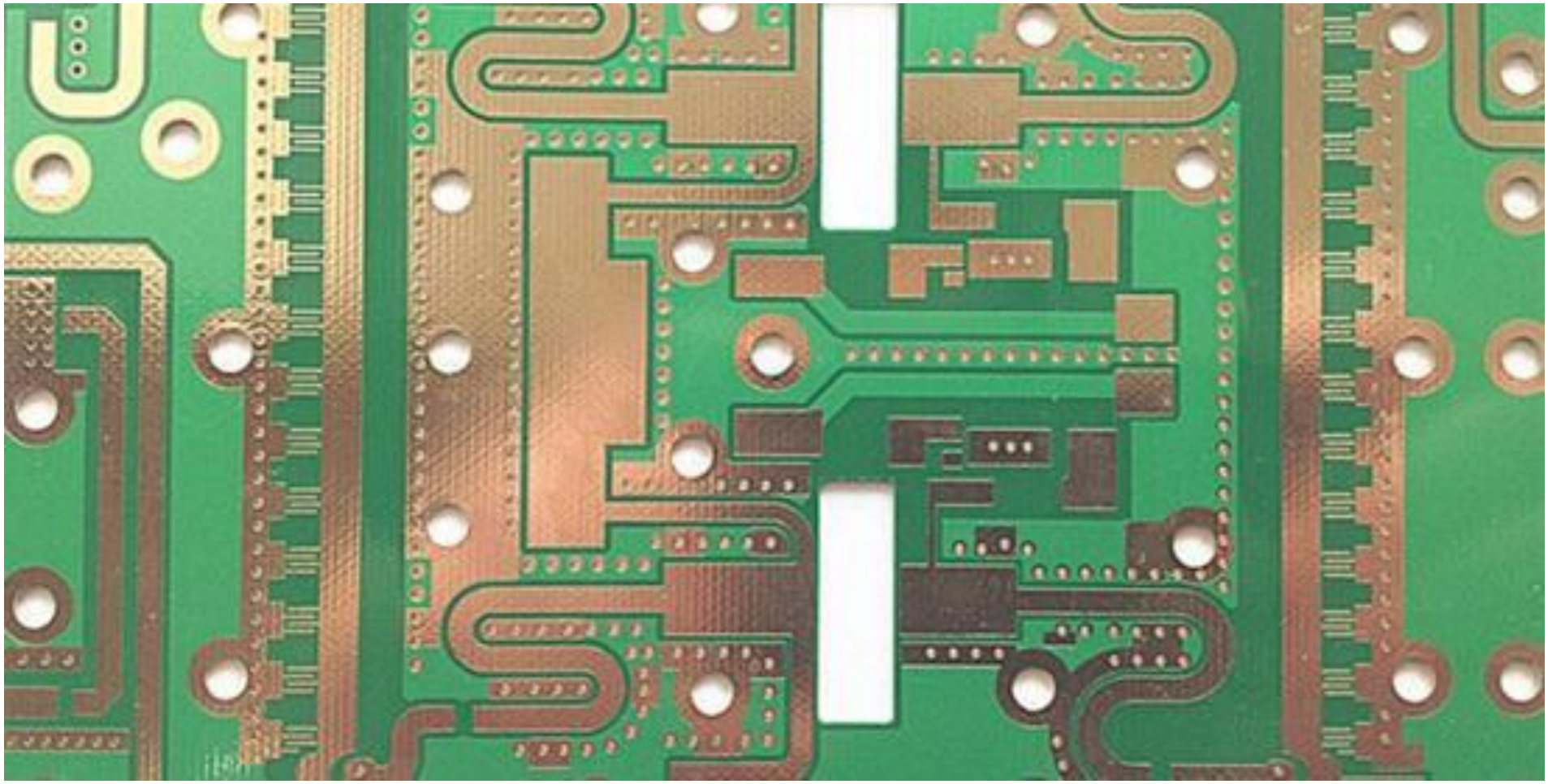
Examples



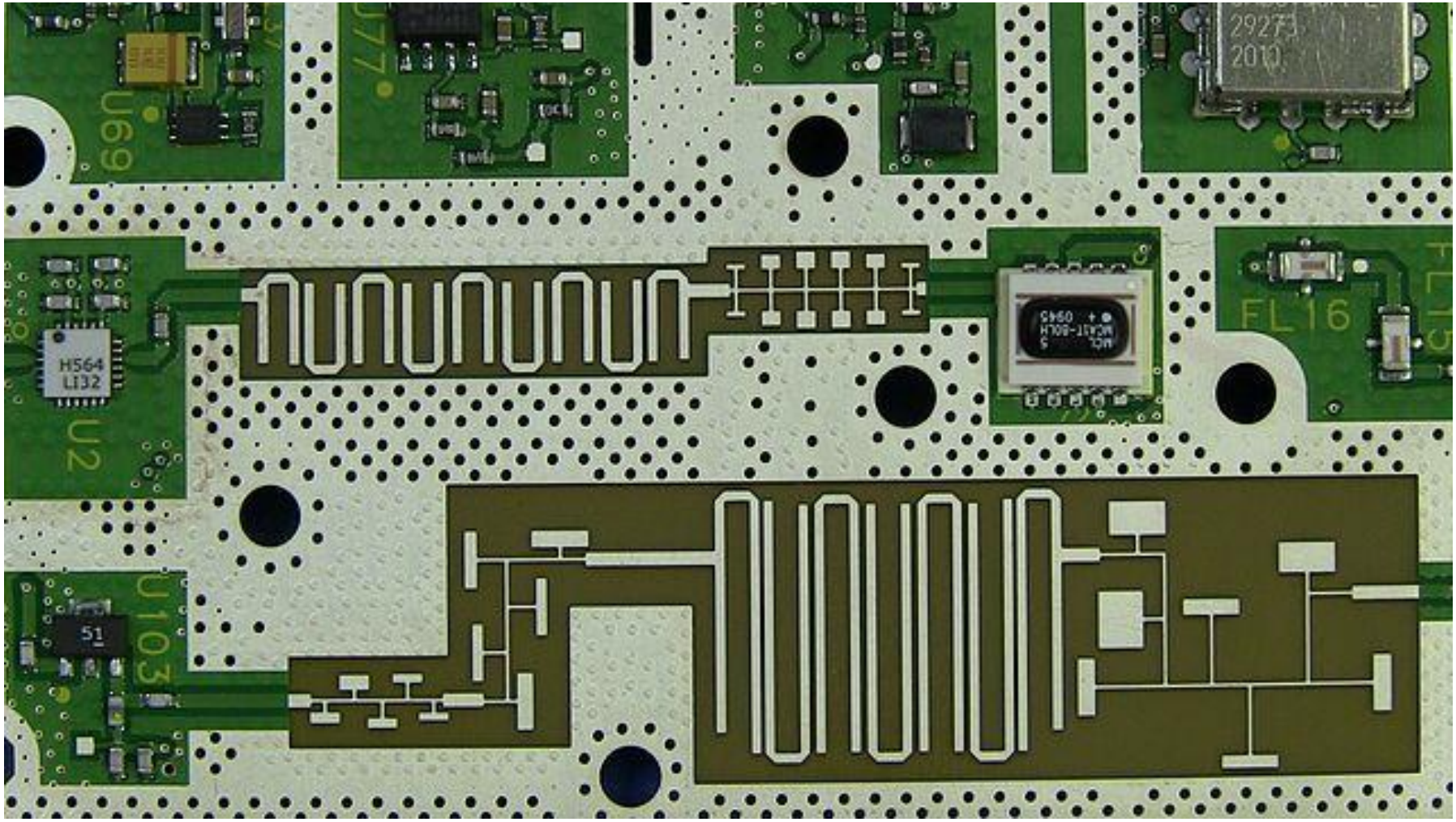
Examples



Examples



Examples



Voltage standing wave ratio

$$V(z) = V_0^+ \cdot \left(e^{-j\cdot\beta\cdot z} + \Gamma \cdot e^{j\cdot\beta\cdot z}\right) \qquad |V(z)| = |V_0^+| \cdot |e^{-j\cdot\beta\cdot z}| \cdot |1 + \Gamma \cdot e^{2j\cdot\beta\cdot z}| \qquad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\cdot\beta\cdot z}|$$

maximum magnitude value for $e^{\theta+2j\cdot\beta\cdot z} = 1$ $\qquad V_{\max} = |V_0^+| \cdot (1 + |\Gamma|)$

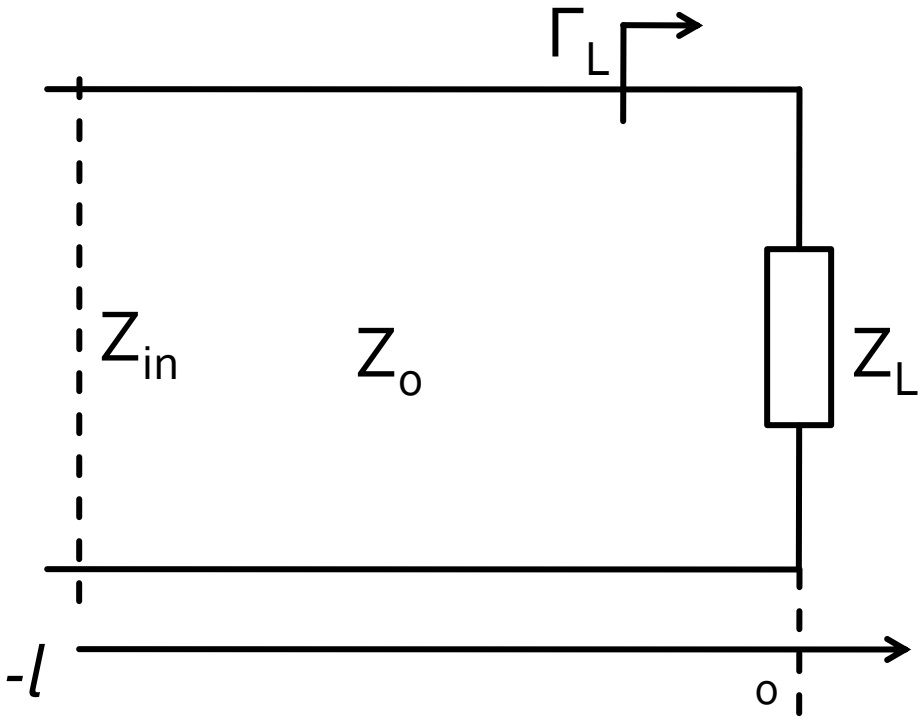
minimum magnitude value for $e^{\theta+2j\cdot\beta\cdot z} = -1$ $\qquad V_{\min} = |V_0^+| \cdot (1 - |\Gamma|)$

- SWR is defined as the ratio between maximum and minimum
  - (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

  - real number $1 \le VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2j \cdot \beta \cdot l}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j \cdot \beta \cdot l}$$

Power transfer

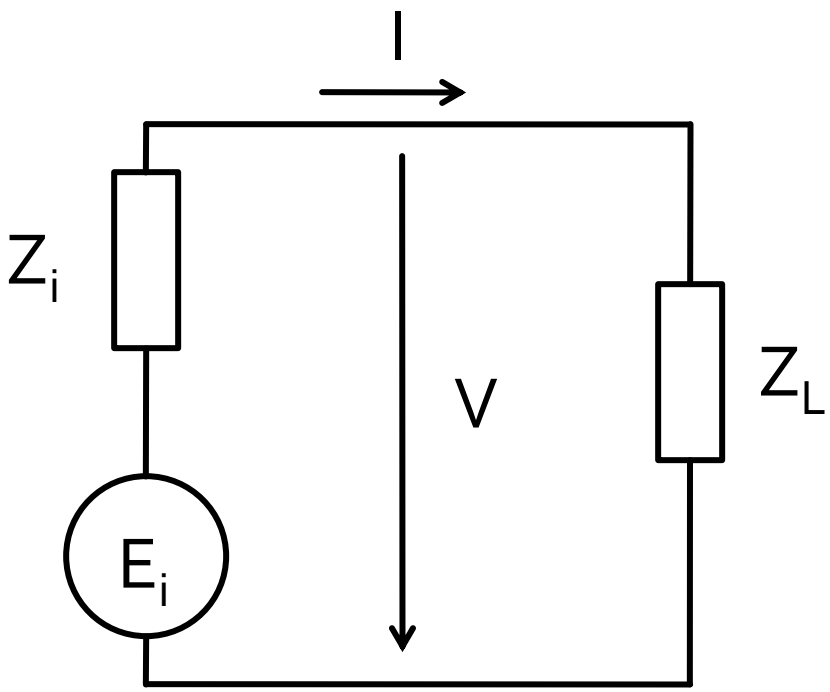
Impedance Matching

Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers~~ ?

Matching

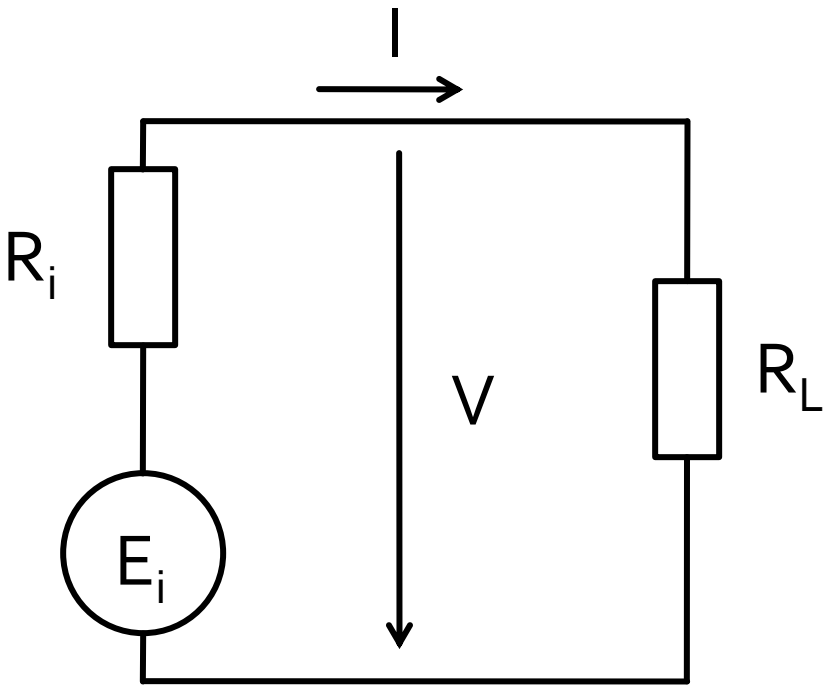
- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

Matching, real impedances

- Source matched to load



$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

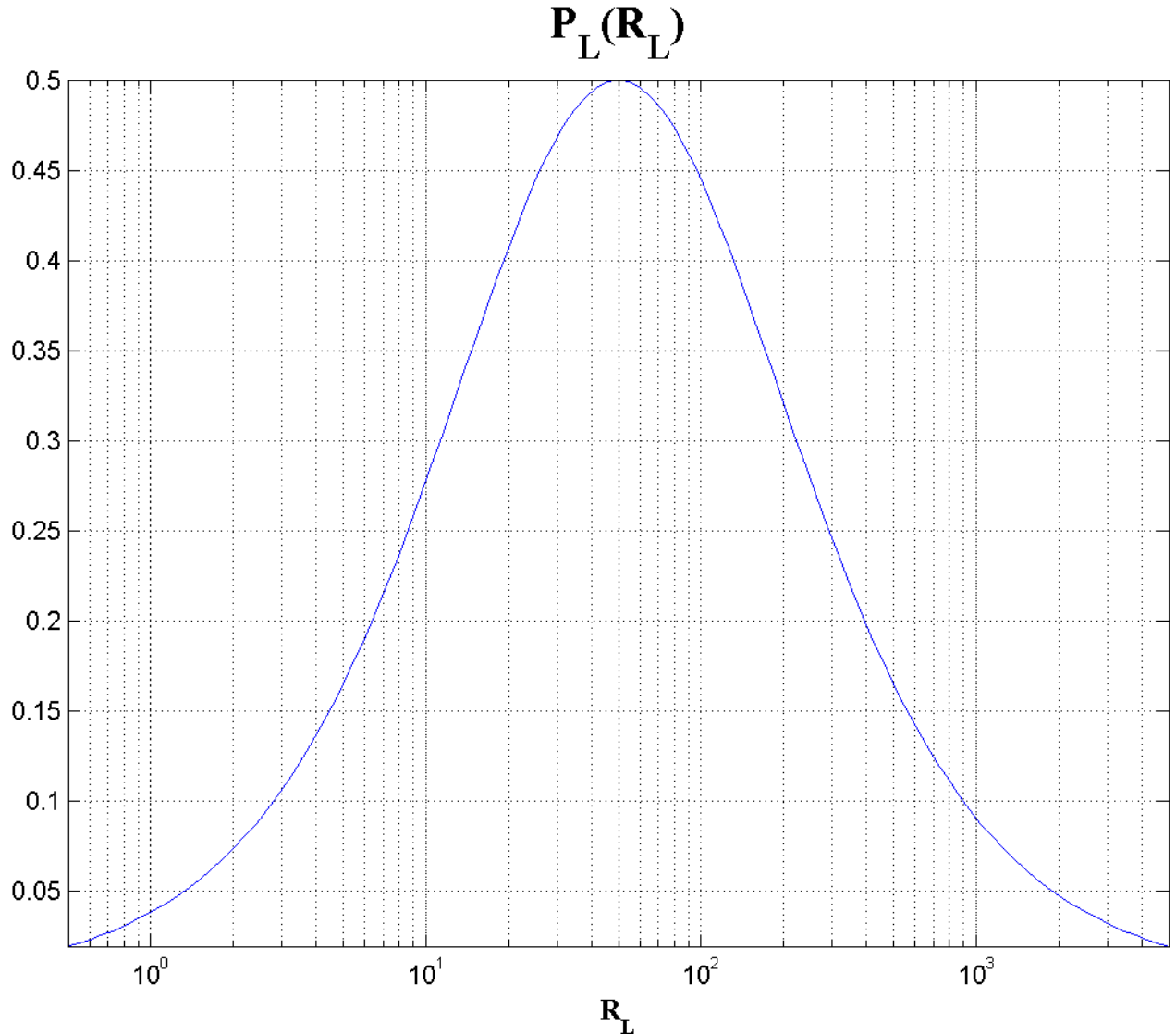
$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

Matching, real impedances

$$P_L = R_L \cdot I^2 \qquad P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

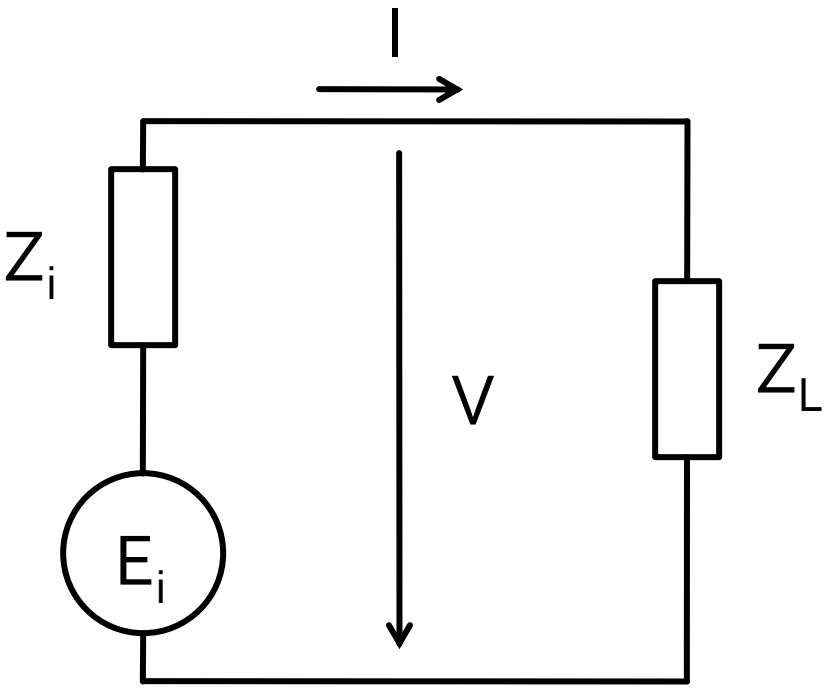
- Power dissipated on load
  - $R_i = 50\Omega$
  - $R_L = 0 \rightarrow P_L = 0$
  - $R_L = \infty \rightarrow P_L = 0$

Matching, real impedances



Matching, complex impedances

- Source matched to load



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \mathrm{Re}\left\{ Z_L \cdot |I|^2 \right\}$$

$$P_L = \mathrm{Re}\left\{ Z_L \right\} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Matching

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

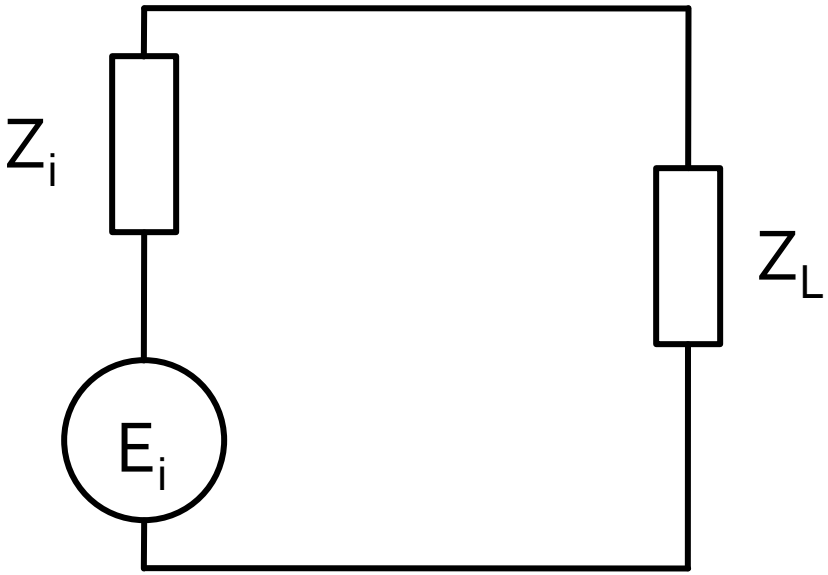
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Matching
  - maximum power transmitted to the load
  - condition?

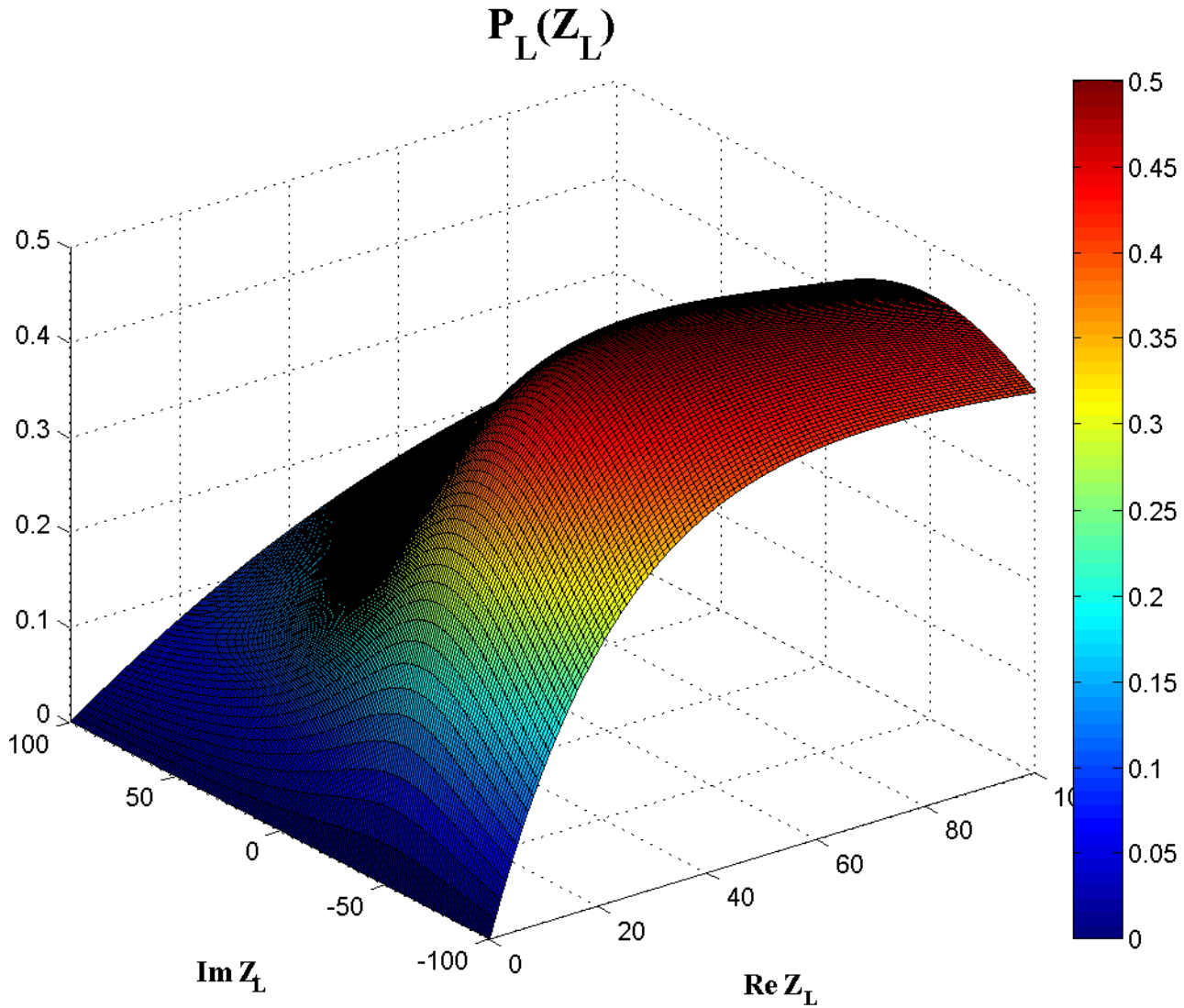
Matching, example

- $E = 10V$
- $Z_i = 50\,\Omega + j \cdot 50\Omega$
- $P_L(Z_L)$?

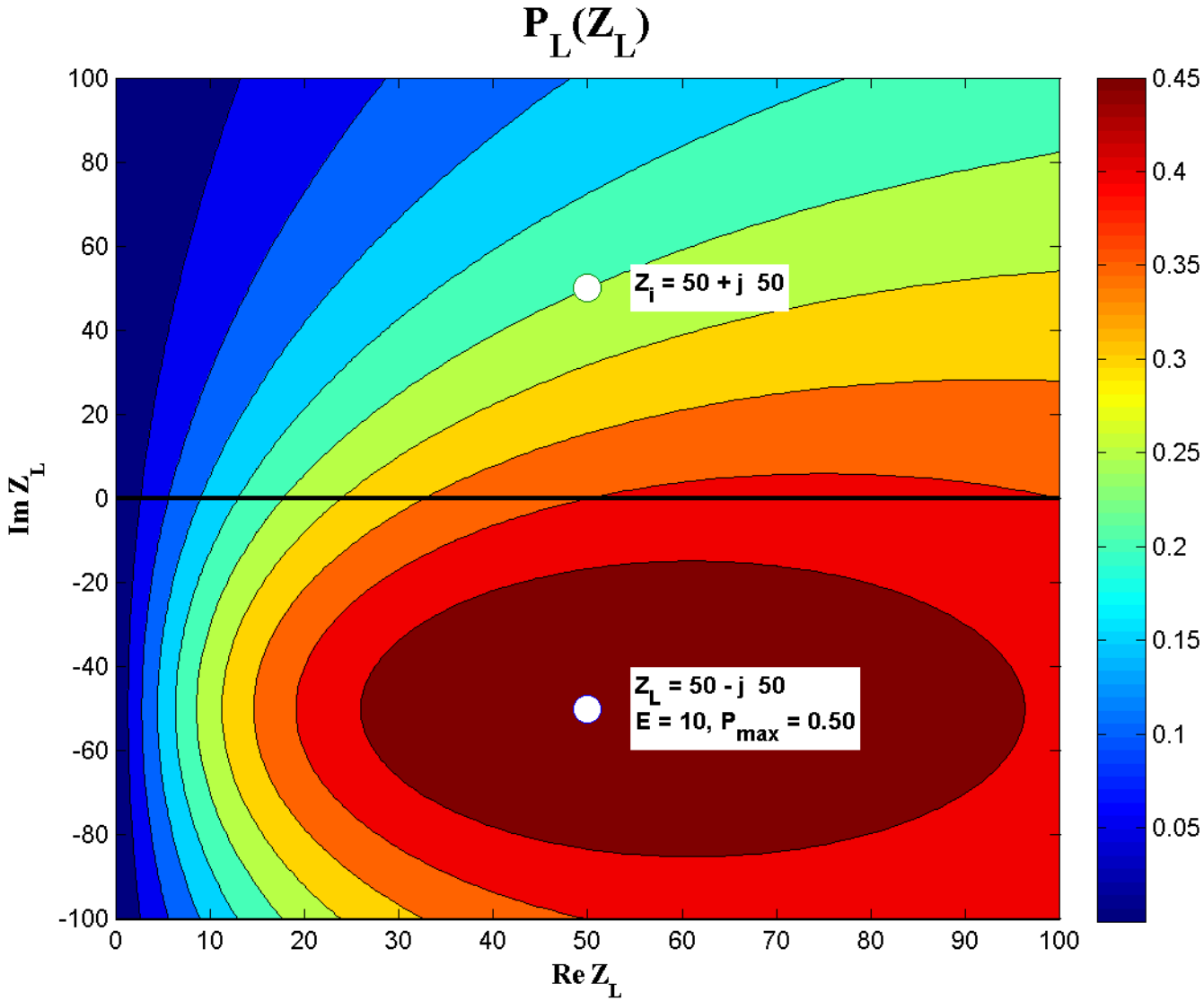
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



Matching, example



Matching, example



Matching , from the point of view of power transmission

$$R_i > 0,\ R_L > 0$$

$$P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a$$

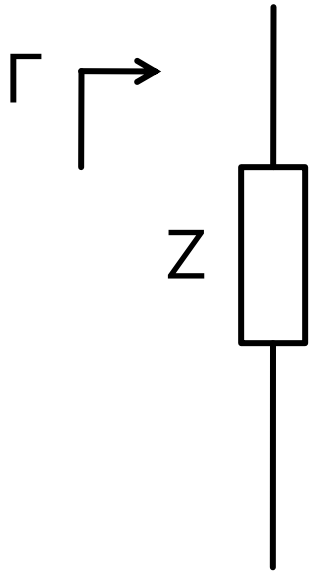
$$R_L = R_i,\ X_L = -X_i$$

- $P_{Lmax} = P_a$: Available Power

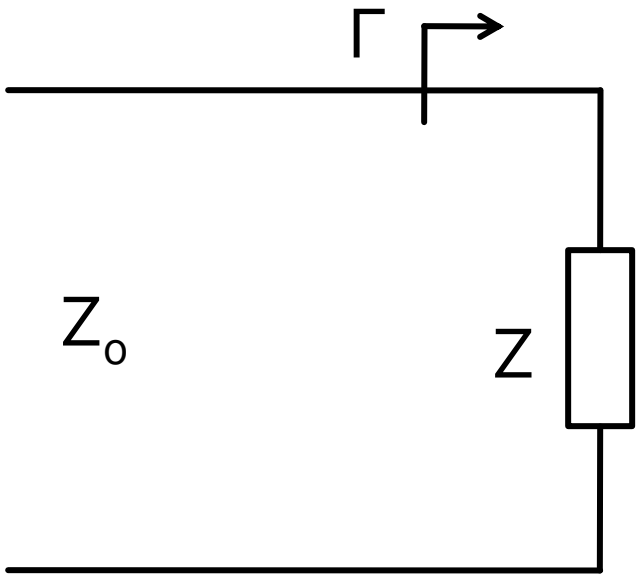
$$Z_L = Z_i^*$$

Reflection coefficient

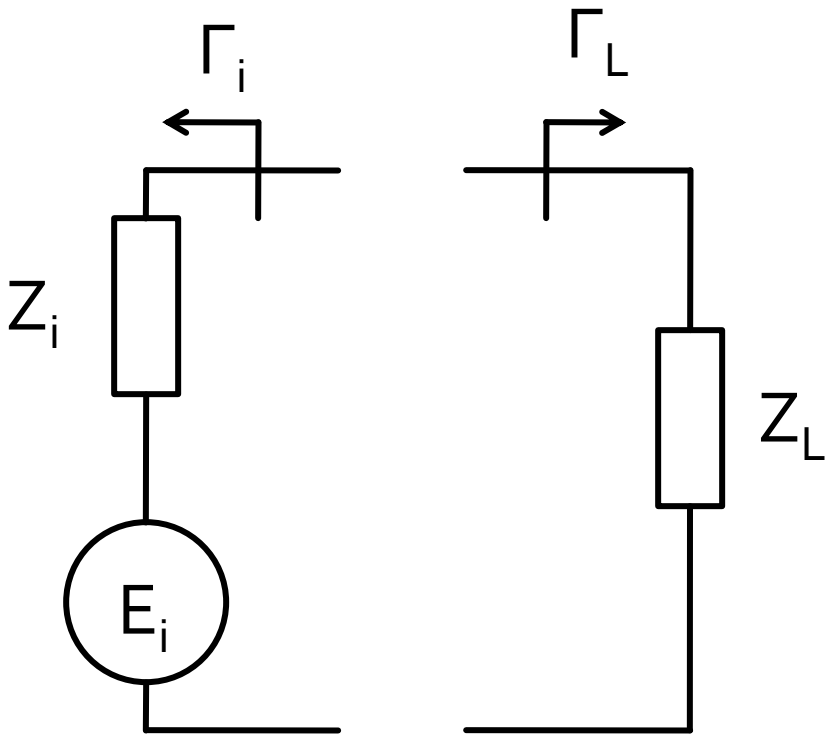
- Any impedance Z_o chosen as reference



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



Matching , from the point of view of power transmission



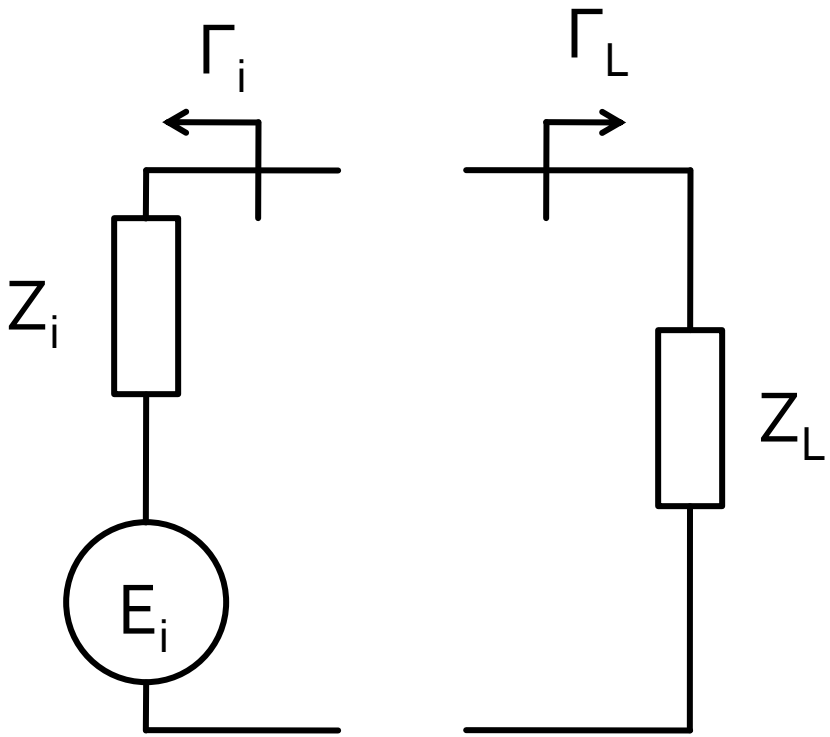
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

Matching , from the point of view of power transmission



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

Matching , from the point of view of power transmission

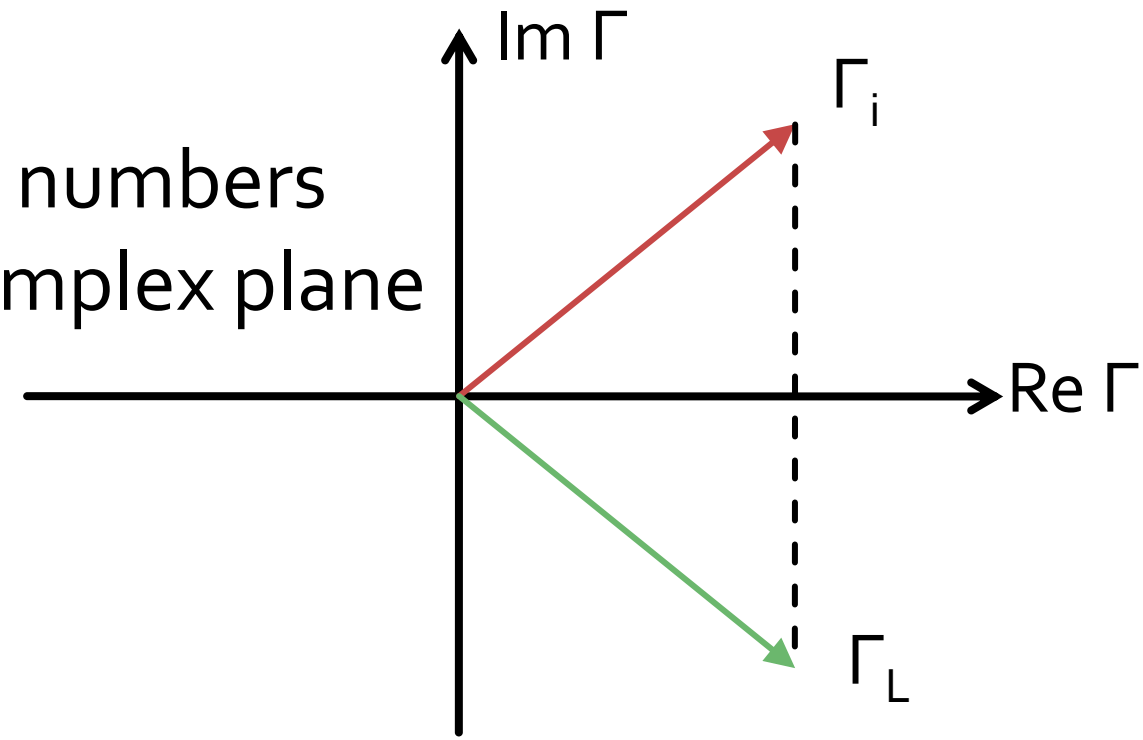
$$Z_L = Z_i^*$$

If we choose a (any) real Zo

$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

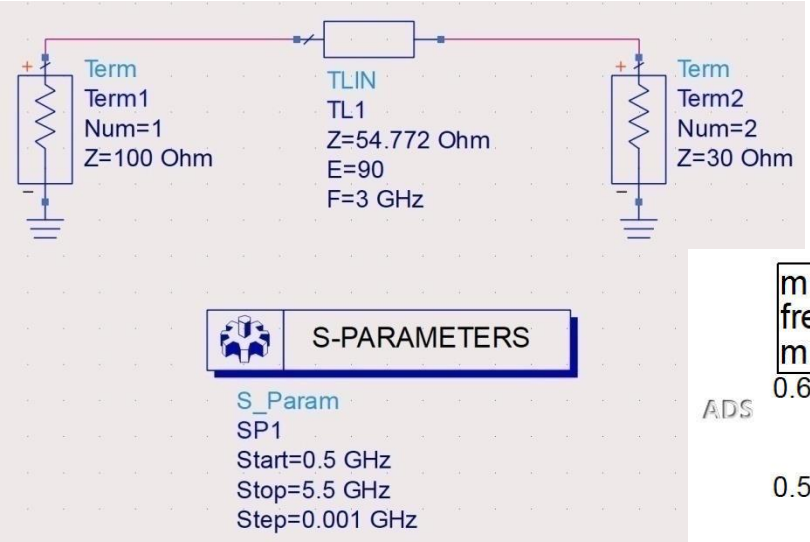
- complex numbers
- in the complex plane

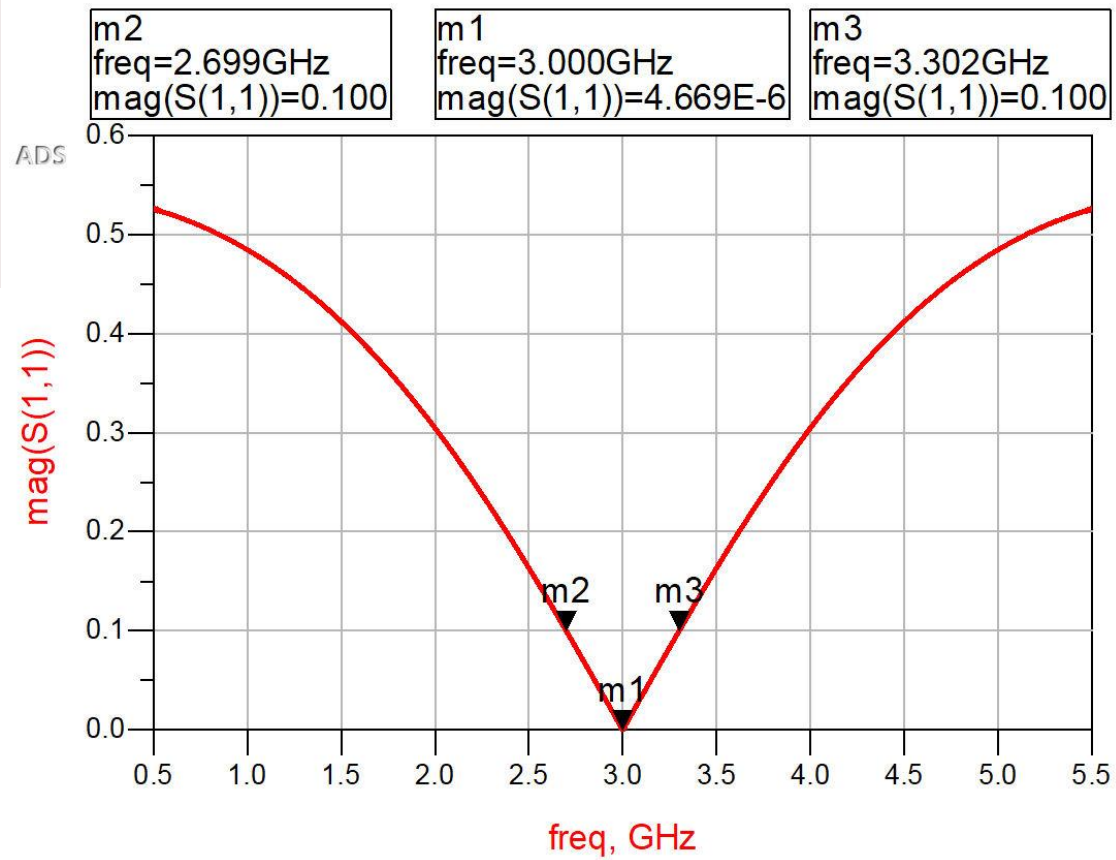


Laboratory 1

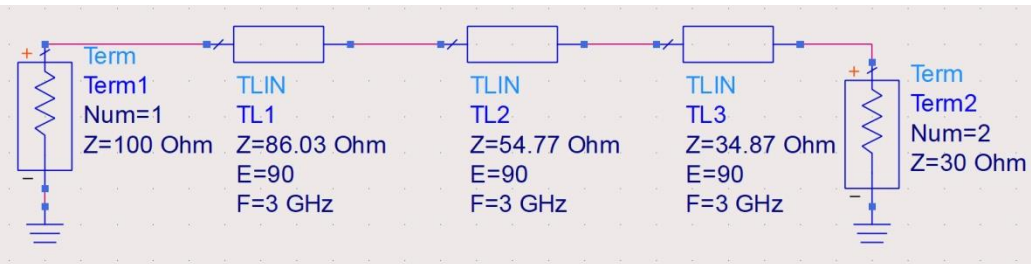
Impedance Matching

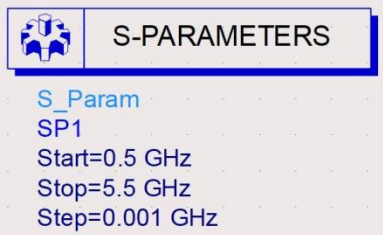
The quarter-wave transformer

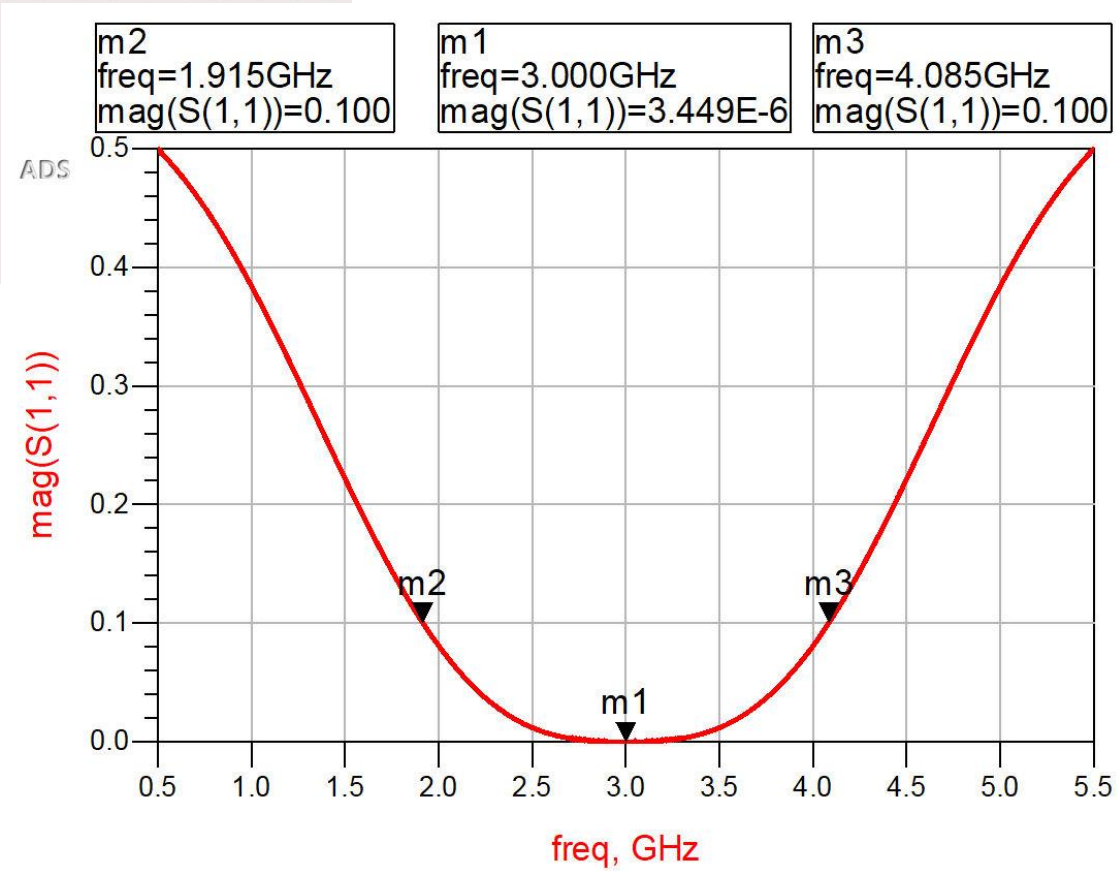




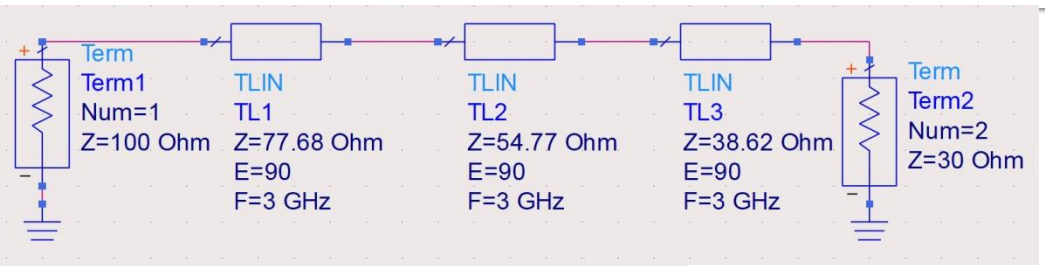
Binomial multisection transformer

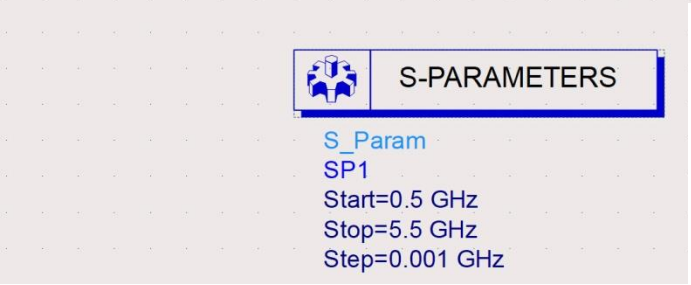


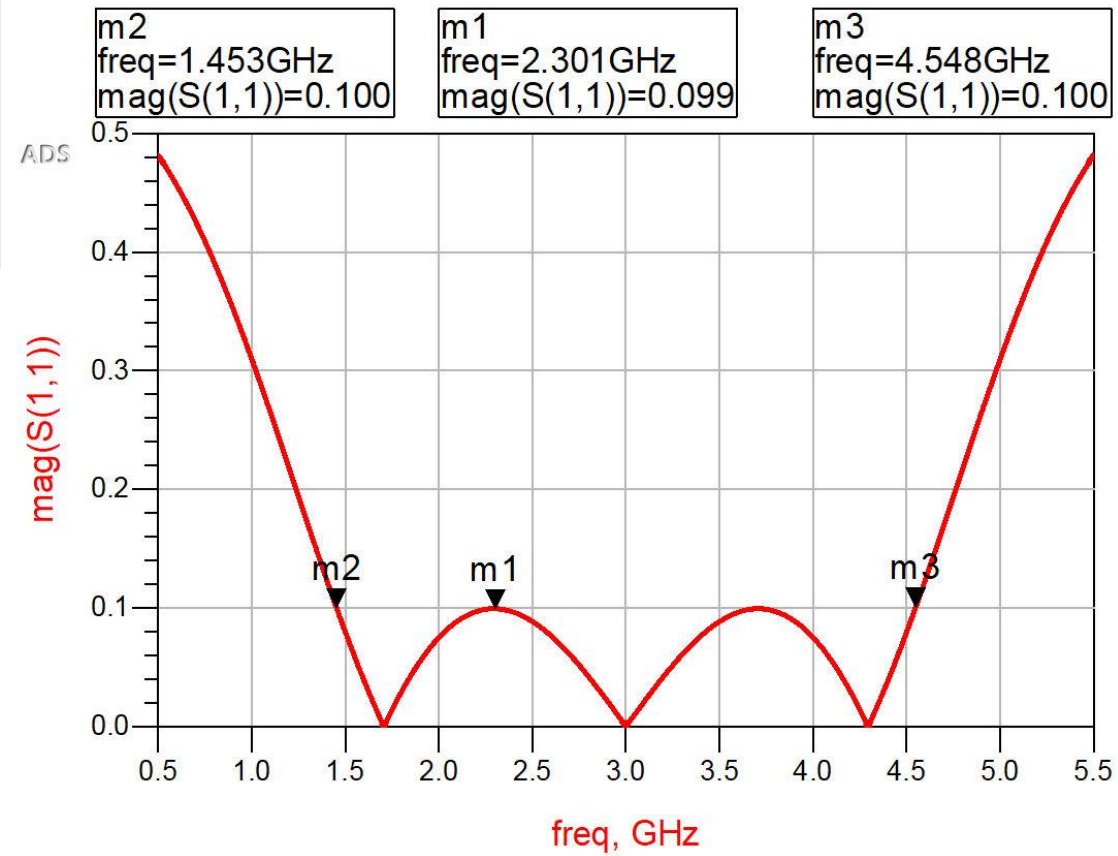




Chebyshev multisection transformer







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